

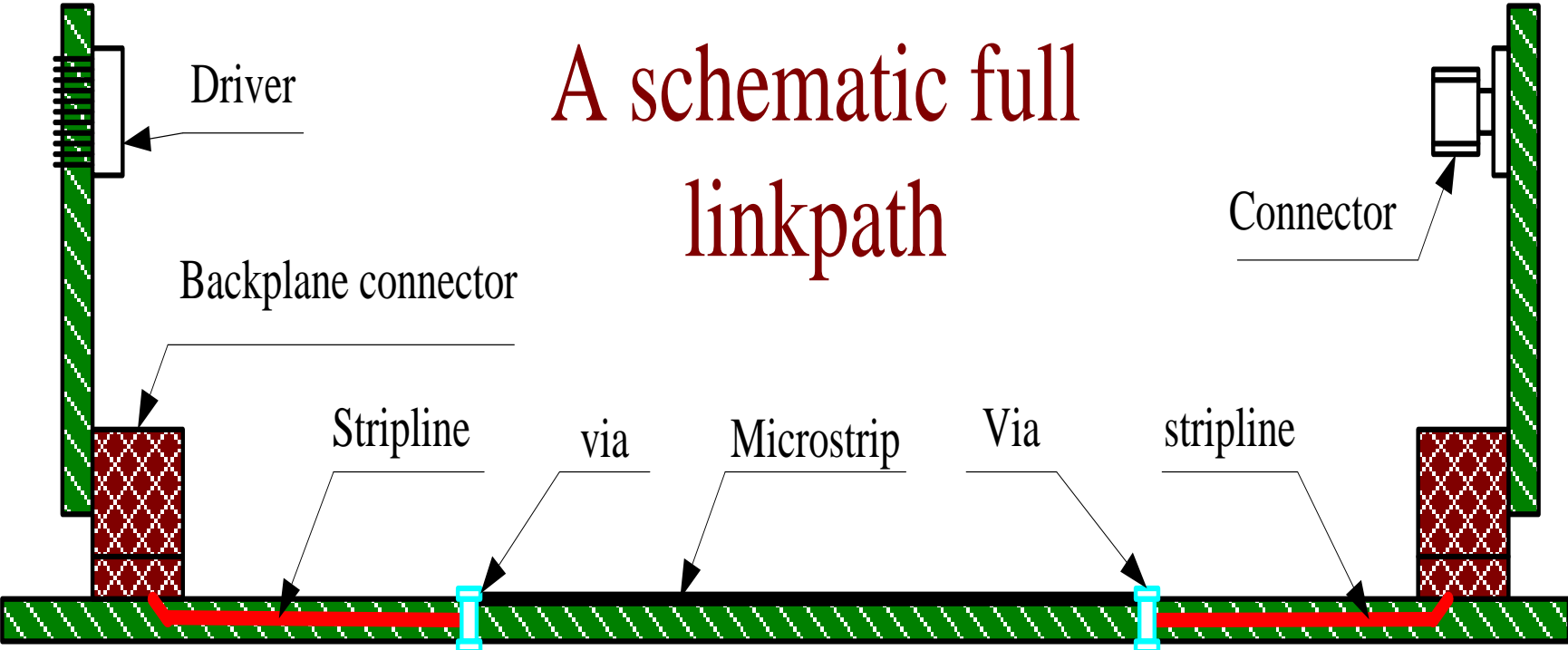
# Recent Via Modeling Methods for Multi-Vias in a Shared Anti-pad

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Electromagnetic Compatibility (EMC) Laboratory,  
Missouri University of Science & Technology

# Signal Link Path

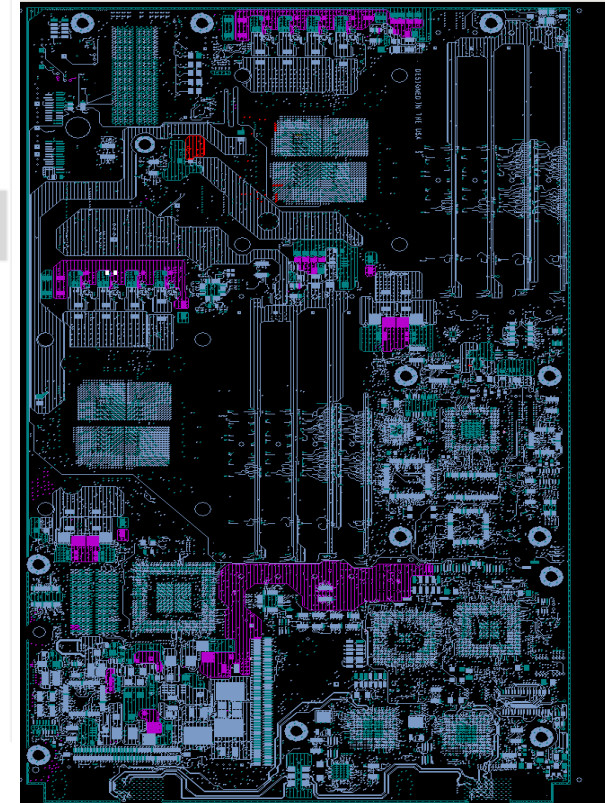
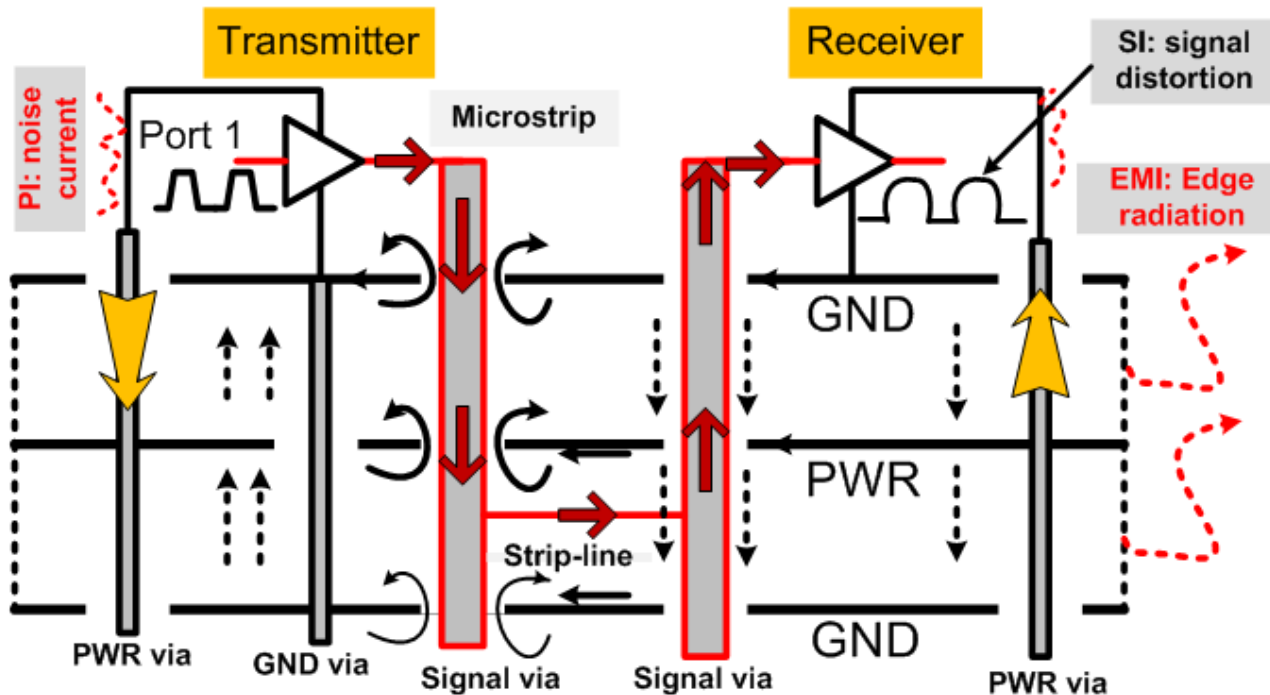
A schematic full linkpath



# Outline

- Vias modeling in signal/power integrity and EMI
- Modeling conventional via structures by hybrid field-circuit via models
  - Physics-based via model
  - Intrinsic via circuit model
- Modeling multi-vias in a shared anti-pad
  - Hybrid 3D/2D finite element methods (FEM)
  - Hybrid FEM and boundary integral equations (BIE)
- Future work

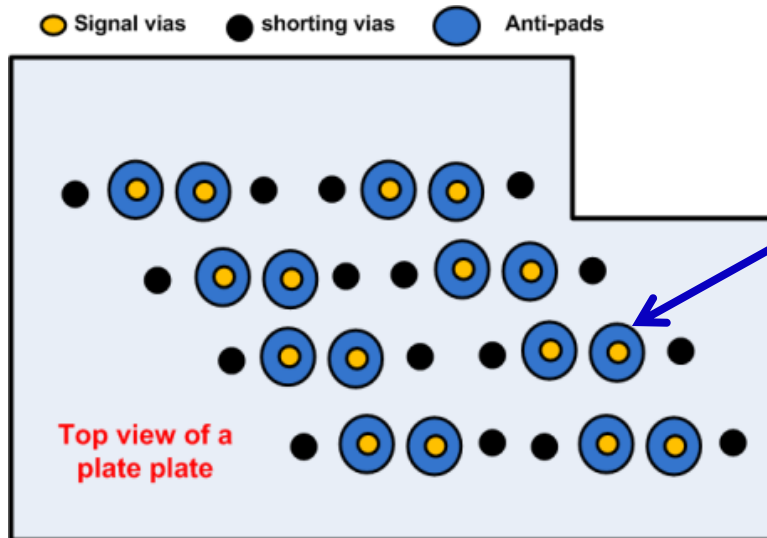
# Via modeling: critical roles in SI, PI & EMI



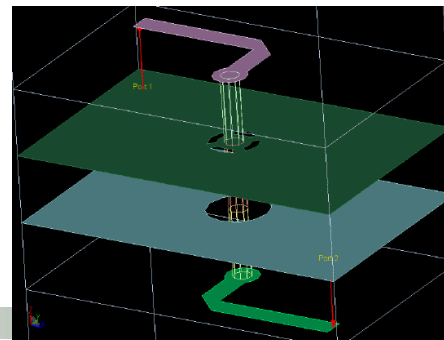
- **SI:** vias are discontinuities for microstrip and strip lines, causing mismatching.
- **PI:** vias excites propagating waves between plate pair, causing noise currents on PWR/GND vias and thus result in voltage fluctuations for I/O buffers.
- **EMI:** propagating waves may lead to plate pair resonances and cause strong edge radiations.

# Overview of via models: a single via in a round anti-pad

- Hybrid field-circuit models
  - Physics-based via model (Prof. Schuster Christian, TUHH);
  - Intrinsic via circuit model (EMC Lab/MST);
- Multiple scattering methods (semi-analytical full-wave method)
  - Conventional multiple scattering method (Prof. Leung Tsang, Univ. Washington);
  - Generalized multiple scattering method (EMC Lab/MST);



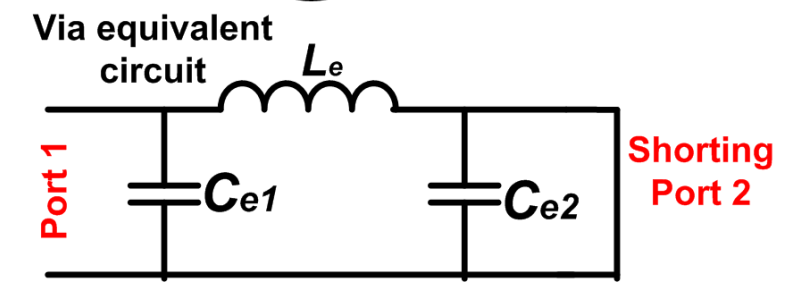
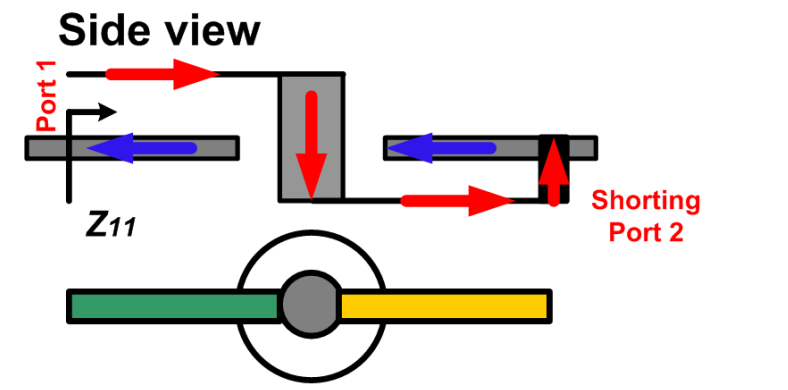
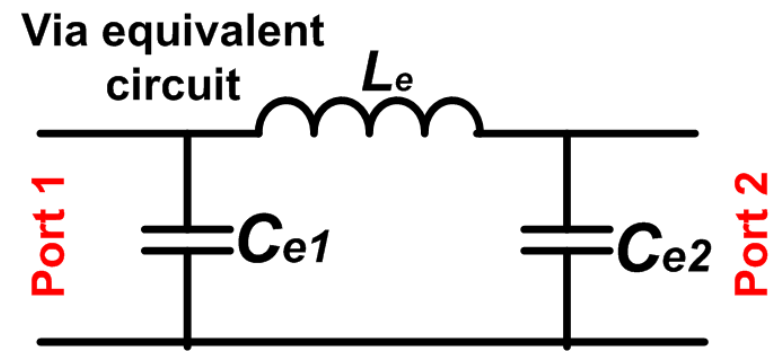
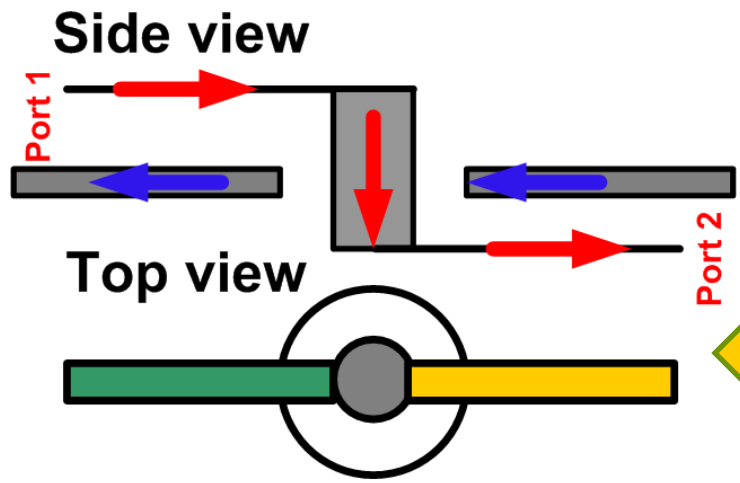
All above methods have an assumption of **coaxial TEM mode in anti-pads in all of via models**



# A via crossing a single plate

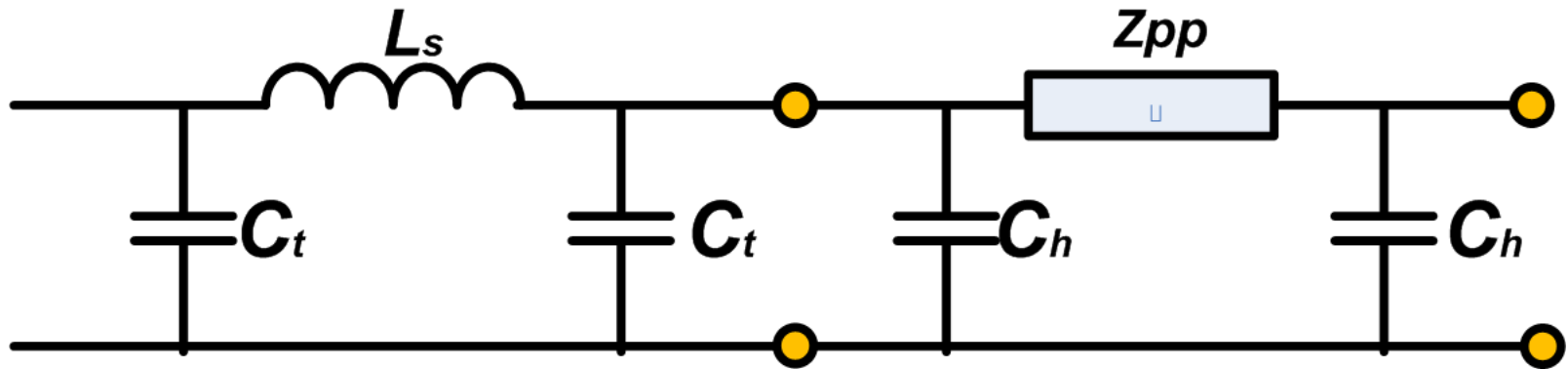
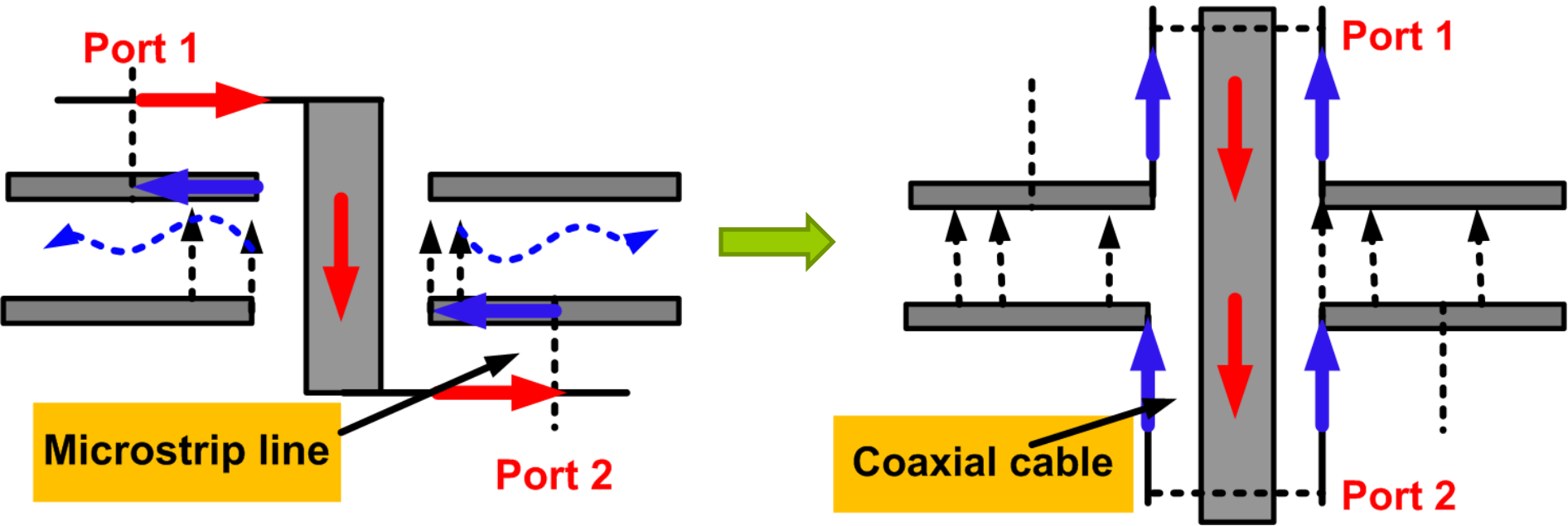
- Full-wave methods (FDTD, MoM)

Quasi-static integral equation method to extract the excess capacitances



By open and shorting ports, we can “guess” a circuit model based on our physics intuition.

# A via crossing a plate pair



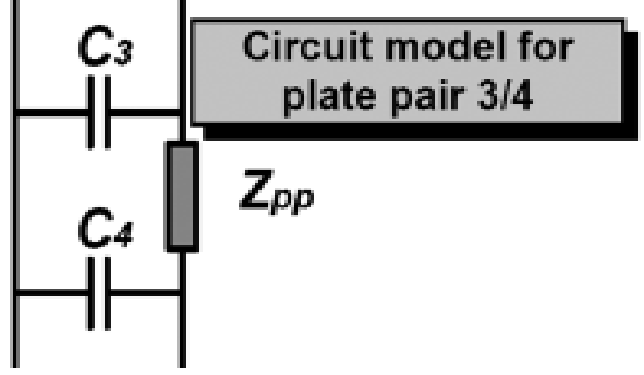
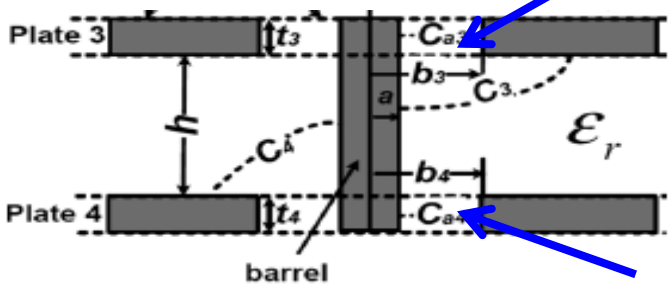
Transmission Line

Physics-based via  
circuit model

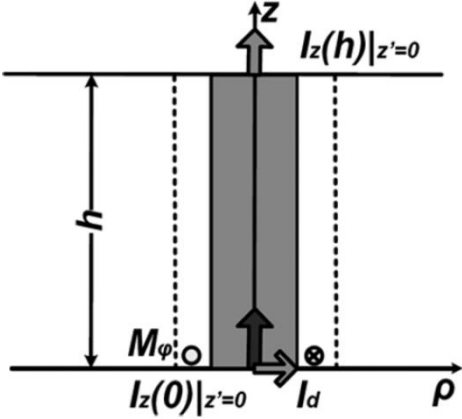
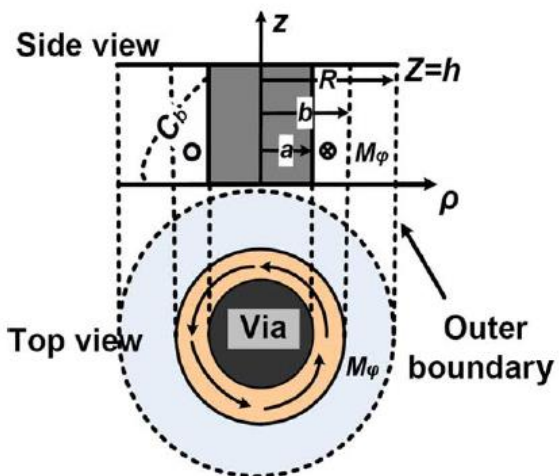


# Physics-based via model: via-plate capacitance

Coaxial TEM mode



Coaxial TEM mode



Analytical formula

$$C_b = \frac{8\pi\epsilon}{h \ln(b/a)} \sum_{n=1,3,5,\dots}^{2N-1} \frac{\left(1 - \Gamma_a^{(n)} \Gamma_R^{(n)}\right)^{-1}}{k_n^2 H_0^{(2)}(k_n a)} \cdot \left\{ \left[ H_0^{(2)}(k_n b) - H_0^{(2)}(k_n a) \right] + \Gamma_R^{(n)} \left[ J_0(k_n b) - J_0(k_n a) \right] \right\}$$

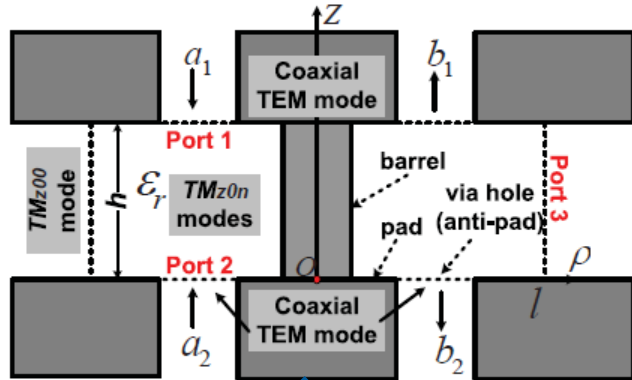
**Magnetic frill current**

**Question:** what contribution from the even-number parallel plate waveguide modes?

Y. Zhang, J. Fan, G. Selli, M. Cocchini, F. D. Paulis, "Analytical evaluation of via-plate capacitance for multilayer printed circuit boards and packages," *IEEE Trans. Microwave Theory Tech.*, vol. 56, no. 9, pp. 2118-2128, Sep 2008.

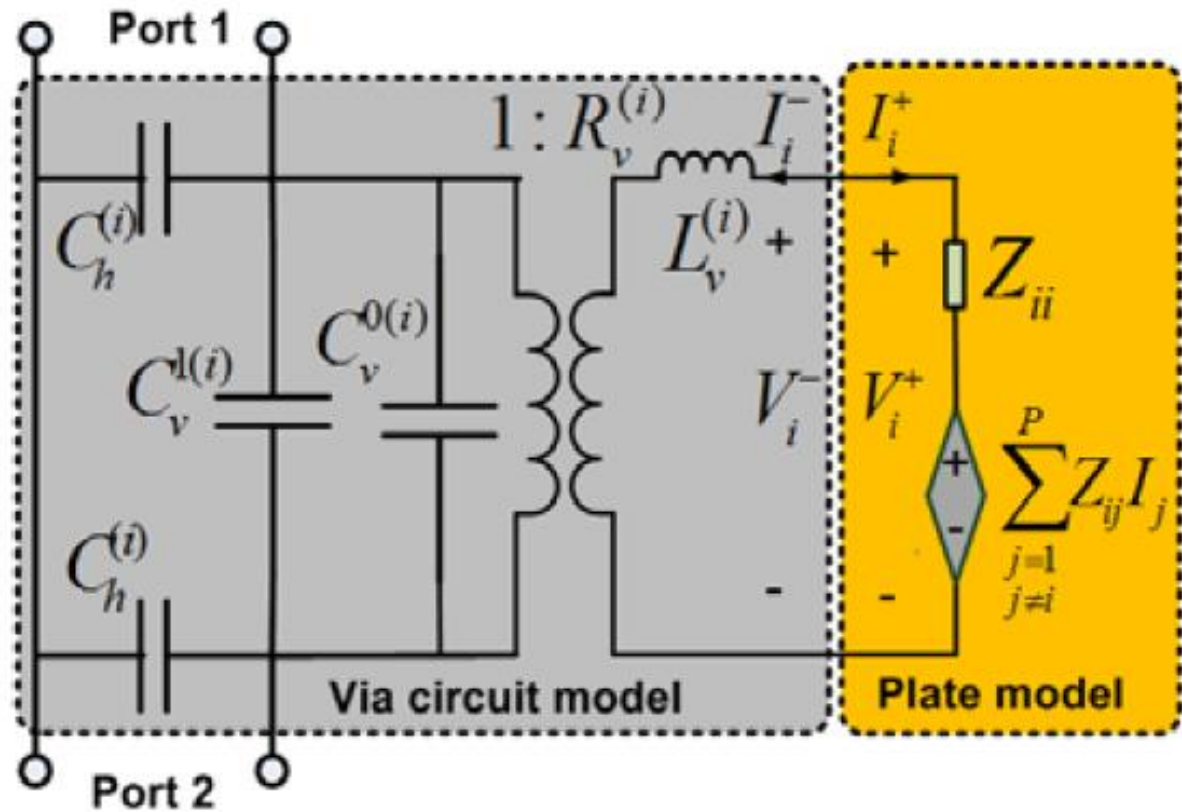


# Intrinsic via circuit model



Localized geometric parameters

Circuit model which satisfies via boundary conditions

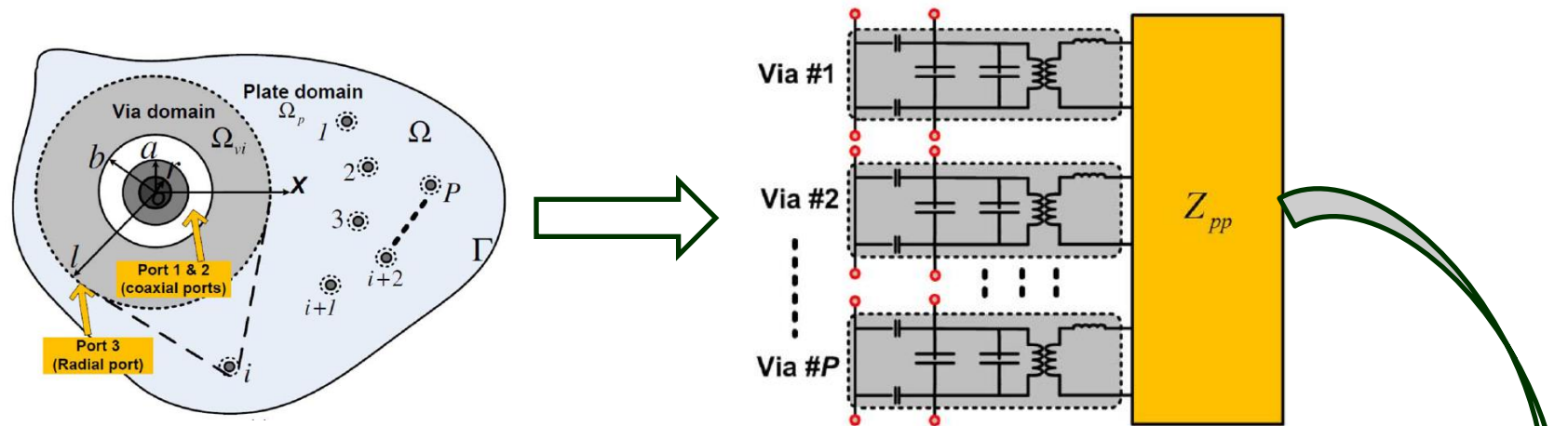


All parasitic parameters are extracted by analytical expression in a fraction of seconds!!

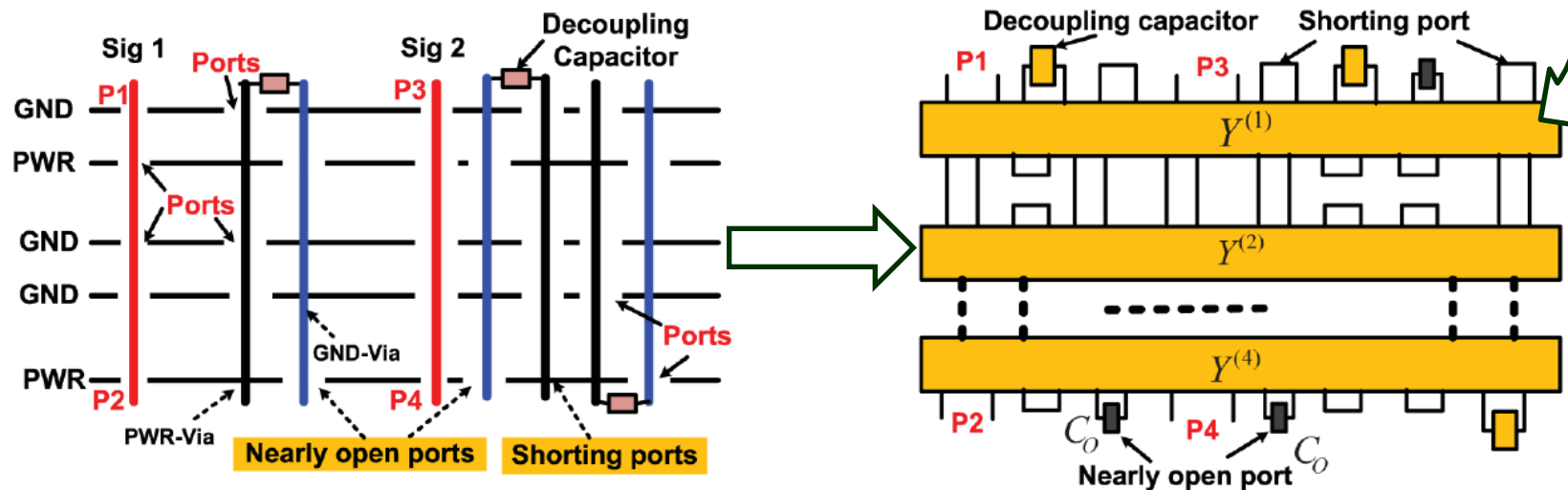
Y. -J. Zhang, and J. Fan, "An intrinsic circuit model for multiple vias in an irregular plate pair through rigorous electromagnetic analysis," *IEEE Trans. Microwave Theory Tech.*, vol. 58, no. 8, pp. 2251-2265, Aug 2010.  
 Y. -J. Zhang, G. Feng, J. Fan, "A novel impedance definition of a parallel plate pair for an intrinsic via circuit model", *IEEE Trans. Microwave Theory Tech.*, vol. 58, no. 12, pp. 3780-3789, Dec 2010.

# Intrinsic via circuit model for SI/PI analysis

## Hybrid field- circuit model for a plate pair



## Systematic modeling multilayer PCB



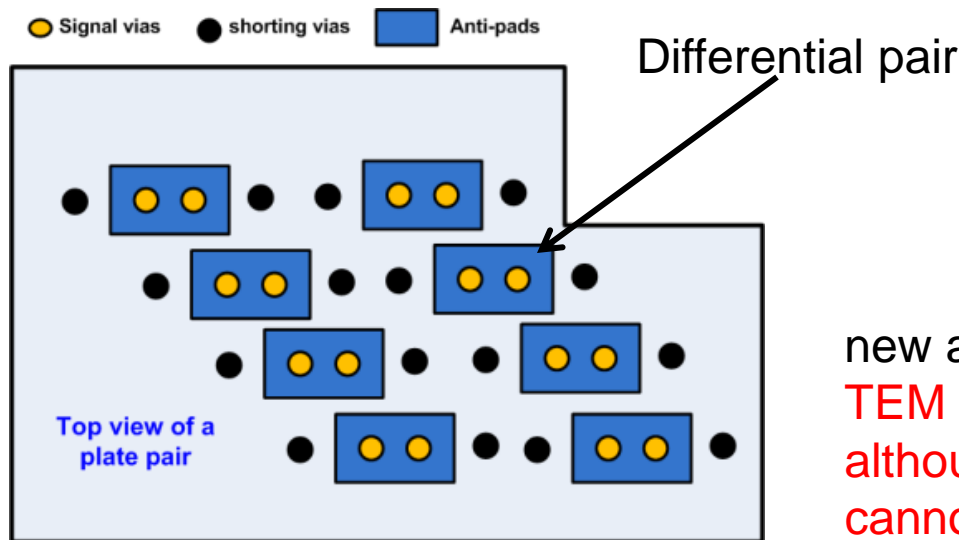
# Multiple vias in a shared anti-pad

## ➤ Hybrid full-wave solvers

- Hybrid 3D/2D finite element method (EMC Lab/MST)
- Hybrid FEM and Boundary integral equations (EMC Lab/MST)

## ➤ (Generalized) multiple scattering method

By Prof. Leung Tsang, Univ. of Washington



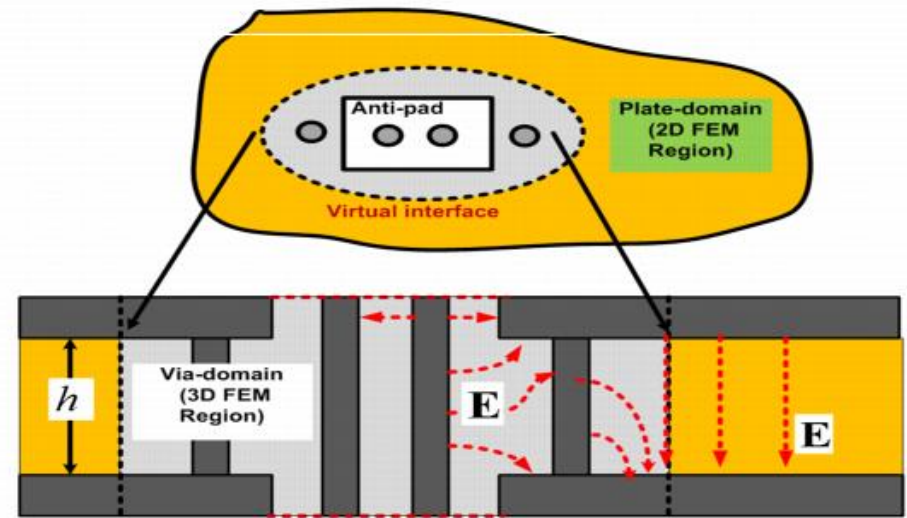
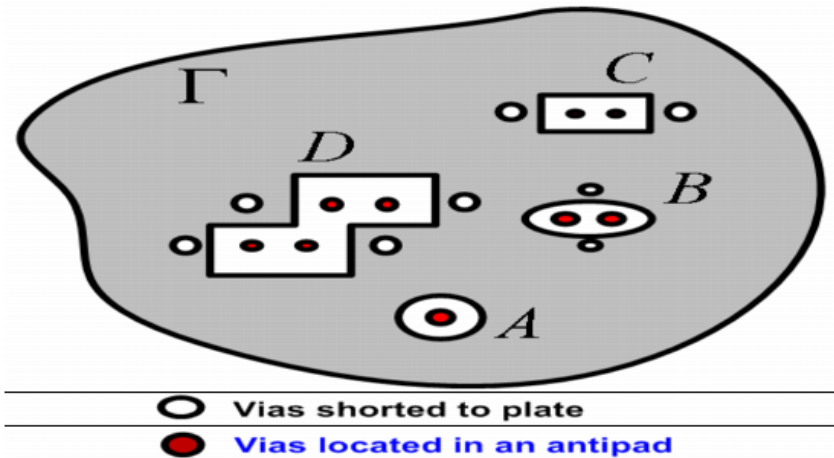
### Challenges for via modeling:

- No analytical field distribution in anti-pads;
- Circuit models difficult to extract.

new assumption:

TEM mode in anti-pads  
although analytical expressions  
cannot be obtained.

# Domain decomposition: via- and plate-domains



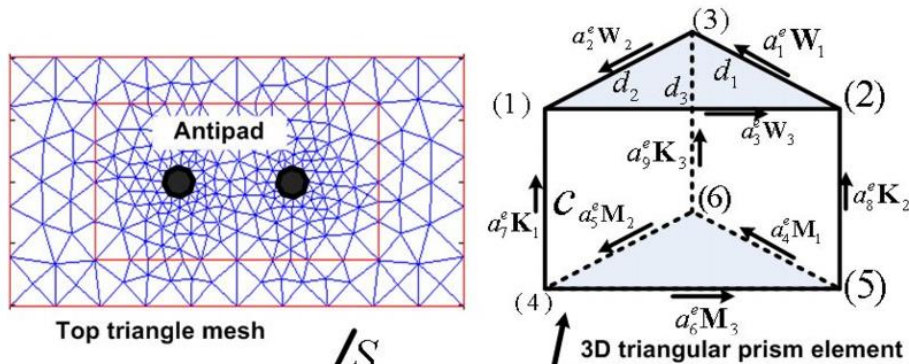
$$k_{\rho} = \sqrt{k_0^2 \epsilon_r - (n\pi/h)^2}. \quad k_0^2 \epsilon_r \ll (\pi/h)^2$$

For  $n$  is not zero, waves will decay exponentially.

Note:

1. A plate pair can be divided by into via-domains and plate-domain(s)
2. via-domains have three-dimensional, complicated field distributions
3. Plate-domain has only TEM mode of parallel plate waveguide (Zero-order parallel plate waveguides)

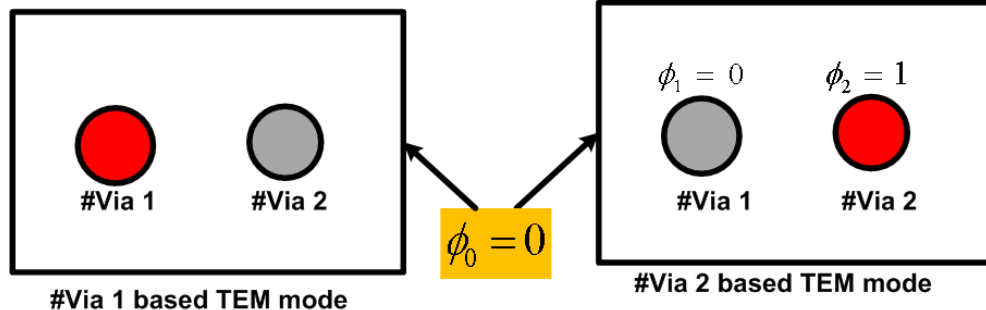
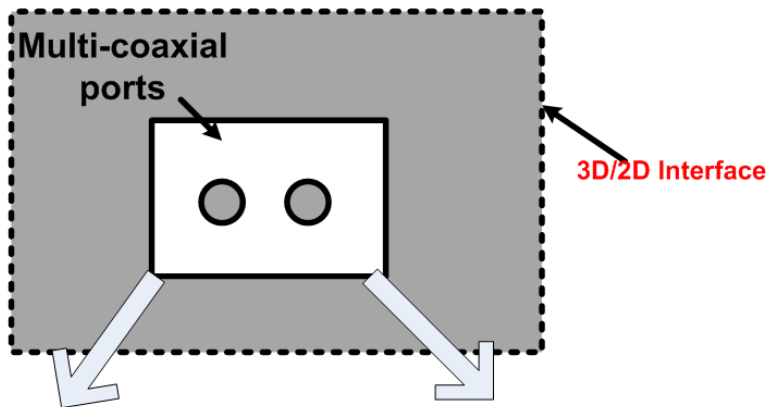
# Hybrid 3D/2D FEM for a plate pair



- 2D electro-static FEM calculation of TEM port modes;
  - A reasonable assumption(?)
  - Used as sources to excite the parallel plate structure Cascaded layer-by-layer
- 3D full-wave FEM for via-domains
  - Higher-order parallel plate modes included
  - Capacitive and inductive coupling among vias
- 2D full-wave FEM for plate-domains
  - Zero-order parallel plate mode
  - Reduced the number of unknowns

How to satisfy the boundary condition at the interface of 3D and 2D FEM regions?

# 2D FEM for TEM modes in an Anti-pad



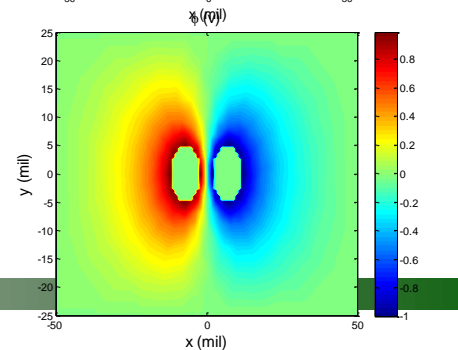
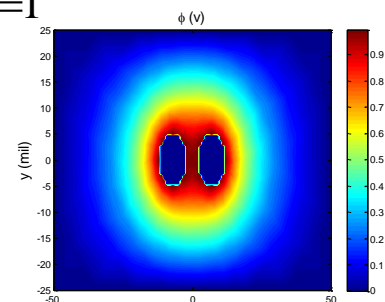
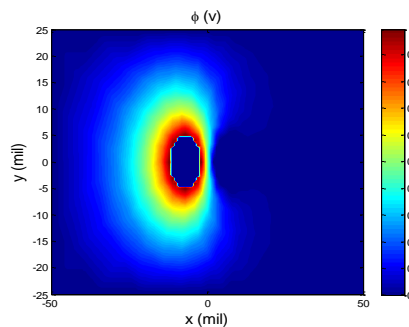
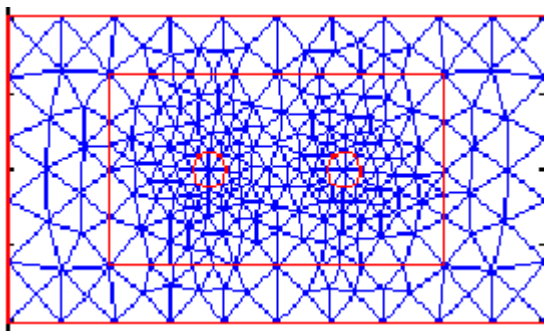
Functional for potential distribution in an antipad

$$F(\phi) = \frac{1}{2} \iint_S \nabla \phi \cdot \epsilon \nabla \phi ds = 0$$

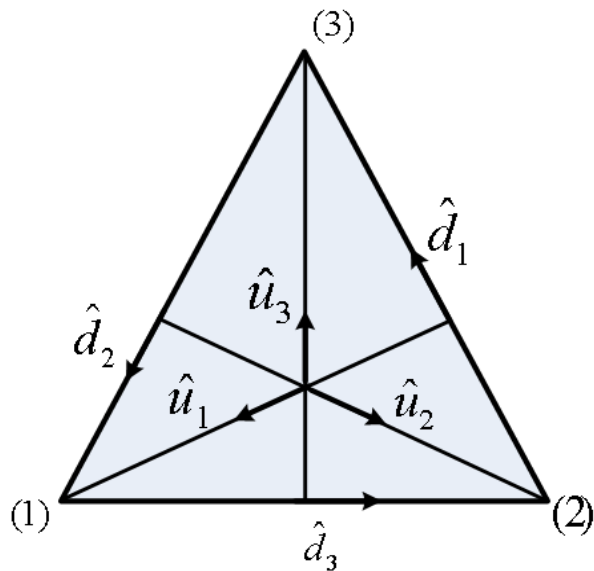
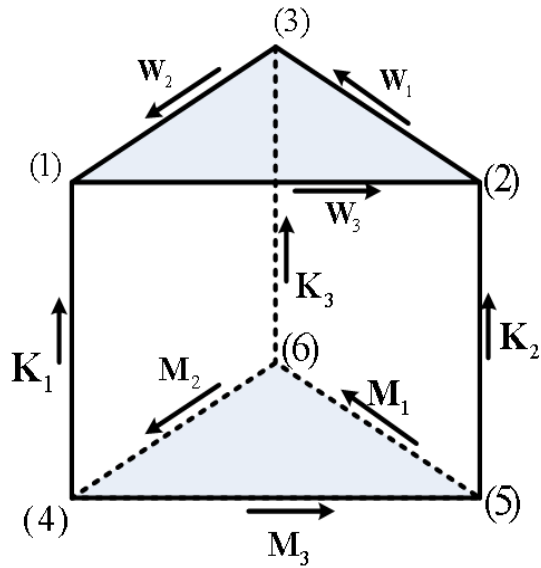
Node-based FEM

$$\phi = \sum_{e=1}^{N_e} \sum_{i=1}^3 a_i^e N_i^e(x, y)$$

Triangular meshes



# Functional of the magnetic fields in 3D FEM



$$F(\mathbf{H}) = \frac{1}{2} \int_V (\nabla \times \mathbf{H} \cdot \epsilon_r^{-1} \nabla \times \mathbf{H} - k_0^2 \mathbf{H} \cdot \mathbf{H}) dV + j\omega\epsilon_0 \oint_S \mathbf{H} \cdot \mathbf{E} \times \mathbf{n} dS$$

$$\mathbf{H} = \sum_{e=1}^{N_e} \sum_{i=1}^9 a_i^e \mathbf{B}_i^e(x, y, z)$$

$$\mathbf{B}_i^e(x, y, z) = \begin{cases} W_i(x, y, z) & i = 1, 2, 3 \\ M_{i-3}(x, y, z) & i = 4, 5, 6 \\ K_{i-6}(x, y, z) & i = 7, 8, 9 \end{cases}$$

TEM modes in anti-pads as sources

$$\mathbf{E} = -\nabla\phi$$

$$W_i = N_i(x, y)z/c$$

$$M_i = N_i(x, y)(1 - z/c)$$

$$K_i = L_i(x, y)\mathbf{e}_z$$

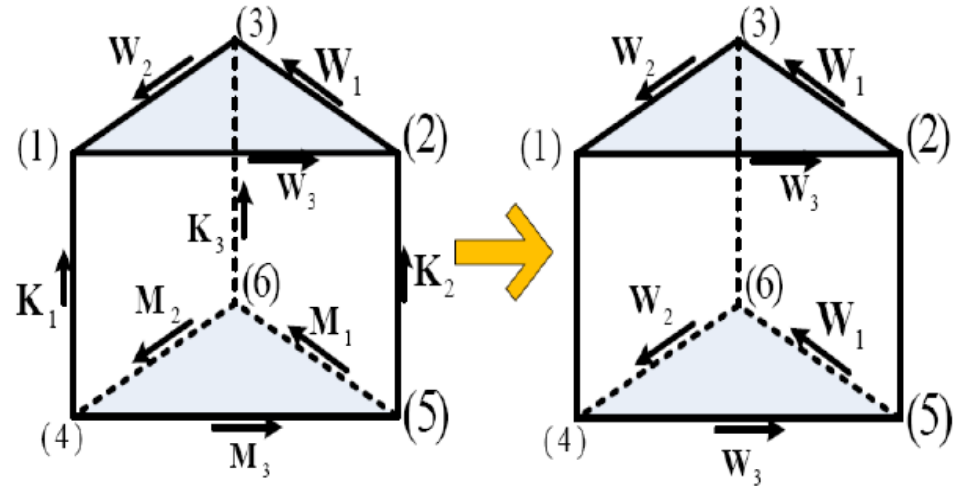
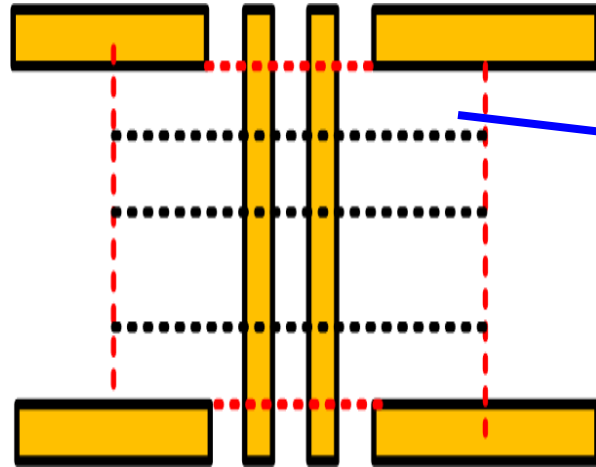
Edge elements

Edge elements

Node elements

$$N_i(x, y) = d_i (L_{i+1} \nabla L_{i+2} - L_{i+2} \nabla L_{i+1})$$

# Triangular Prism Element: 3D $\rightarrow$ 2D (1)



3D Triangular prism element

2D Triangular prism element

using

$$\begin{aligned} a_1^e &= a_4^e & a_7^e &= 0 \\ a_2^e &= a_5^e & a_8^e &= 0 \\ a_3^e &= a_6^e & a_9^e &= 0 \end{aligned}$$

**a 2D triangular prism element !**

**Note:** In coding, these conditions can be automatically satisfied if top and bottom edges in a prism are numbered as same edge numbering index and side-edges are not accounted at all.

3D FEM can be converted into 2D FEM easily and boundary conditions along 2D and 3D domains are satisfied automatically.



# Triangular Prism Element: 3D → 2D (2)

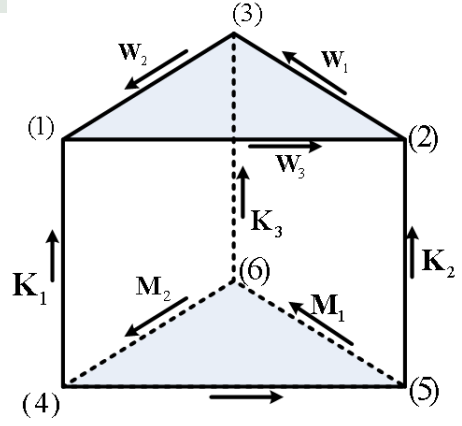
$$K_{ij}^e = \int_{V_e} [\epsilon_r^{-1} \nabla \times \mathbf{B}_i^e \cdot \nabla \times \mathbf{B}_j^e - k_0^2 \mathbf{B}_i^e \cdot \mathbf{B}_j^e] dV_e$$



Add them together.

3D FEM 9-by-9 element matrix

$$K^e = \epsilon_r^{-1} \begin{bmatrix} \left( \frac{c}{3} N_{\nabla} + \frac{1}{c} N \right) & \left( \frac{c}{6} N_{\nabla} - \frac{1}{c} N \right) & -N_{\nabla L} \\ \left( \frac{c}{6} N_{\nabla} - \frac{1}{c} N \right) & \left( \frac{c}{3} N_{\nabla} + \frac{1}{c} N \right) & N_{\nabla L} \\ -N_{\nabla L}^T & N_{\nabla L}^T & cF \end{bmatrix} - k_0^2 \begin{bmatrix} \left( \frac{c}{3} N \right) & \left( \frac{c}{6} N \right) & 0 \\ \left( \frac{c}{6} N \right) & \left( \frac{c}{3} N \right) & 0 \\ -0 & 0 & cE \end{bmatrix}$$



$$\begin{aligned} a_1^e &= a_4^e & a_7^e &= 0 \\ a_2^e &= a_5^e & a_8^e &= 0 \\ a_3^e &= a_6^e & a_9^e &= 0 \end{aligned}$$

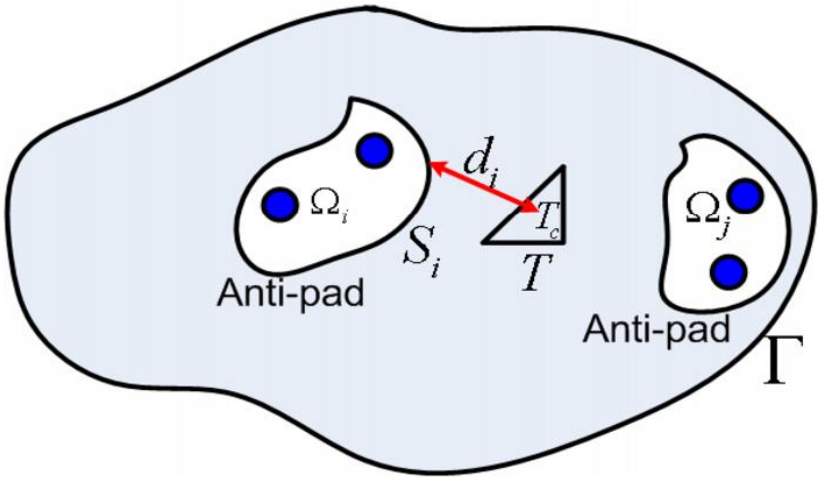
2D FEM 3-by-3 element matrix

Delete the third column and row.

$$K^e = \epsilon_r^{-1} c N_{\nabla} - k_0^2 c N$$

**9 unknowns have been reduced to 3 unknowns!**

# Interface between via-domains and plate-domain



the anti-pad  $\Omega_i$ 's outer contour  $S_i$

The distances between the triangle T and the anti-pads' outer contour.

$$d_i, i = 1, 2, \dots, P$$

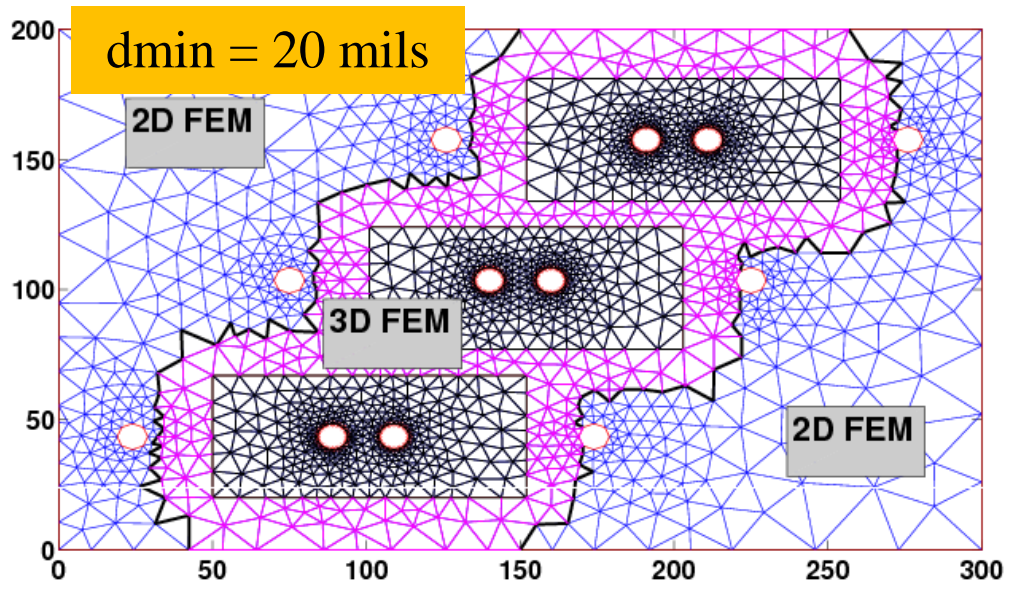
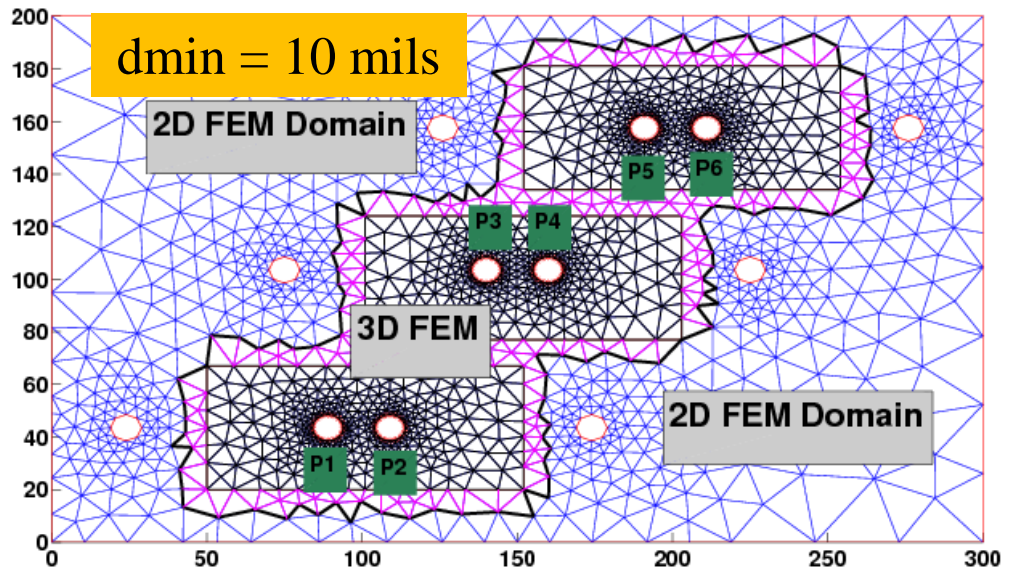
The criterion distance,  $d_{min}$ , depends mainly on the operating frequency, the dielectric constant of the material between the two plates and the plate separation,  $h$ , as the horizontal wave number of the parallel plate mode is

## Transverse Wavenumber Parallel-plate modes

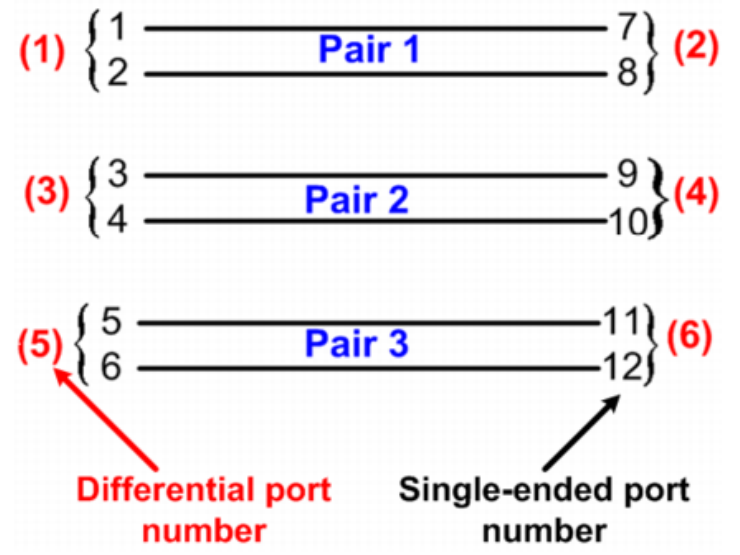
$$k_\rho = \sqrt{k_0^2 \epsilon_r - (n\pi/h)^2} \quad \longrightarrow \quad e^{-\pi \rho/h} \quad \text{for } n=1$$

First higher-order mode decays away from the anti-pad.

# Different criterion distance

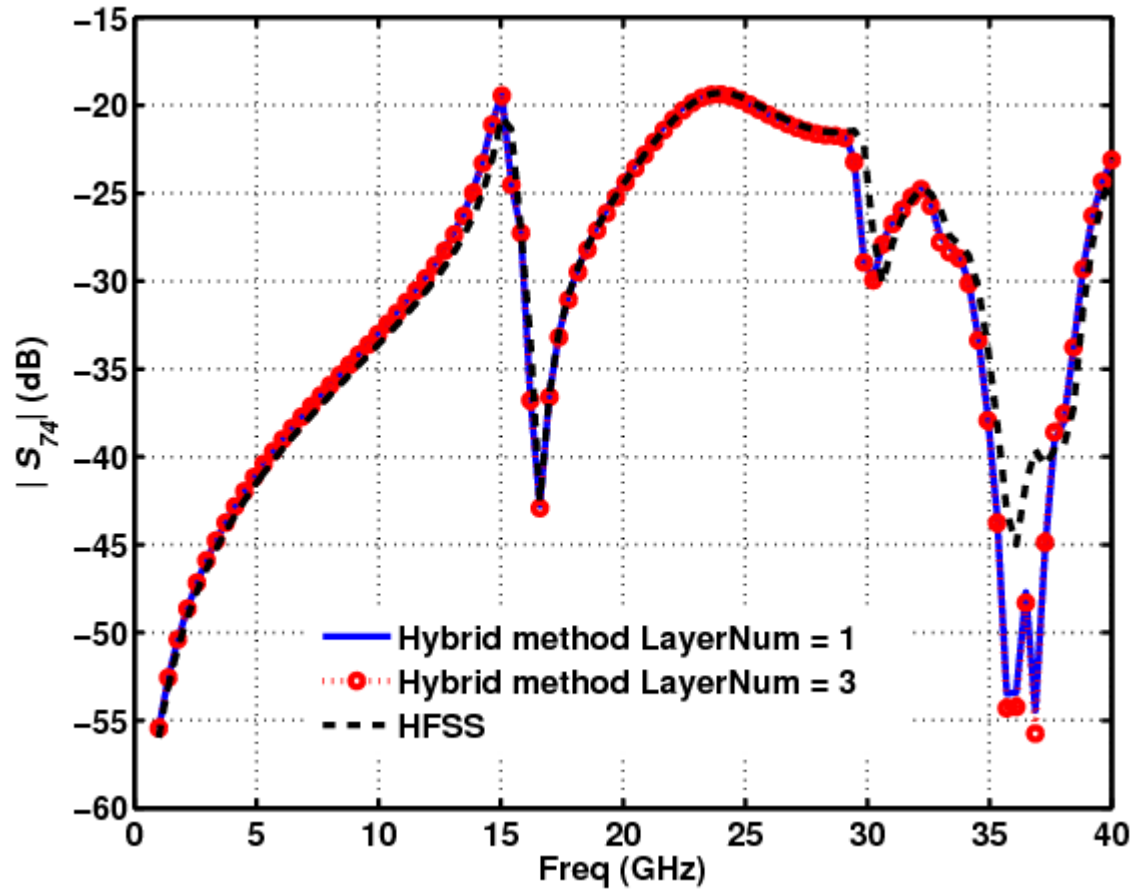
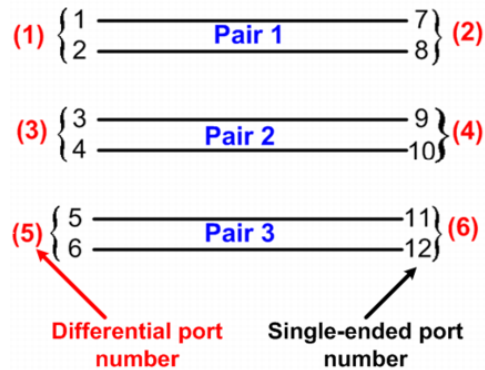
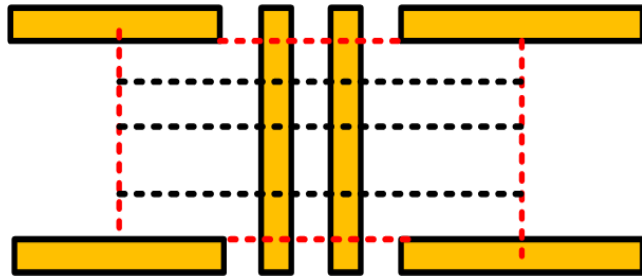


Unit: mil = 0.0254 mm  
 $h = 10$  mils,  $t = 1$  mil  
 $\epsilon_{psr} = 4.0$ ,  $\tan\text{Loss} = 0.02$



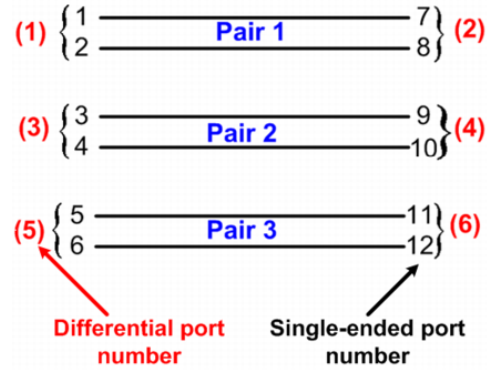
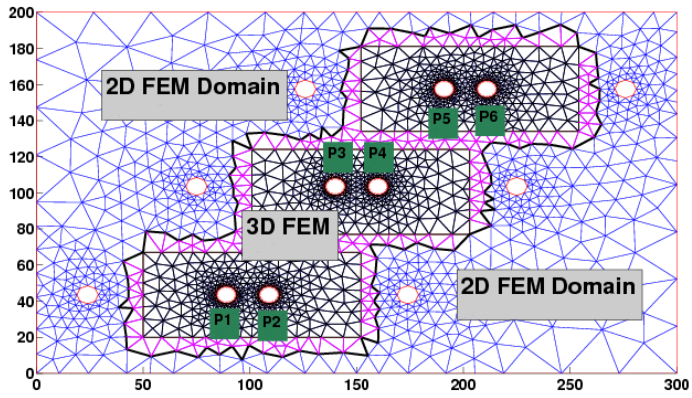
Note:  
**Our experience indicates  $d_{min} = 1\sim 3 h$  is accurate enough.**

# Layer number of triangular prisms

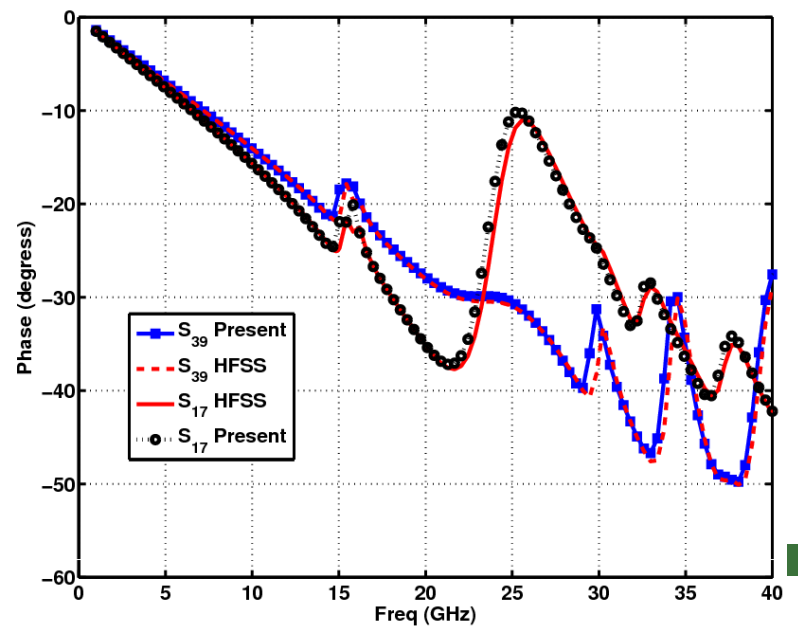
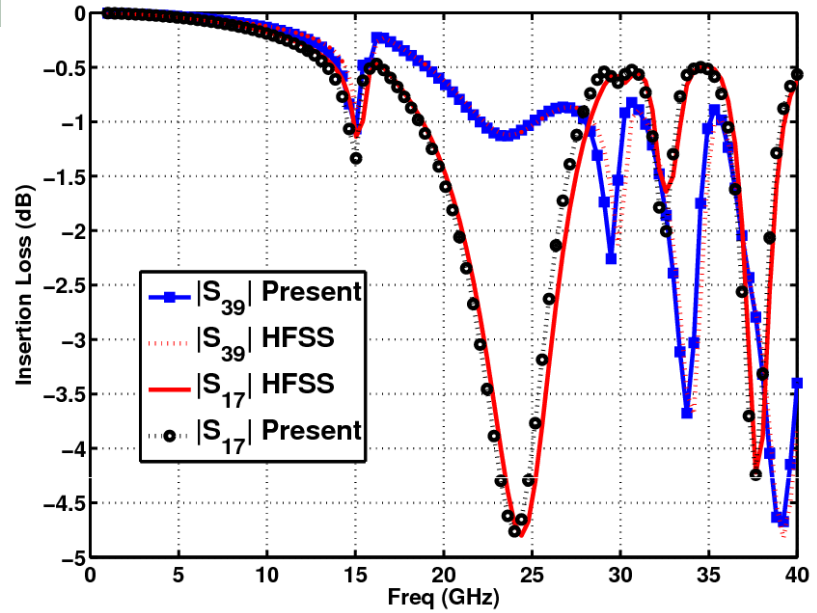


1. The hybrid method has been validated by comparing with HFSS
2. Surprisingly, only one layer of prisms is good enough.

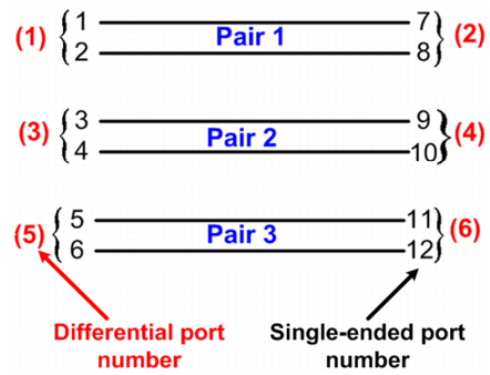
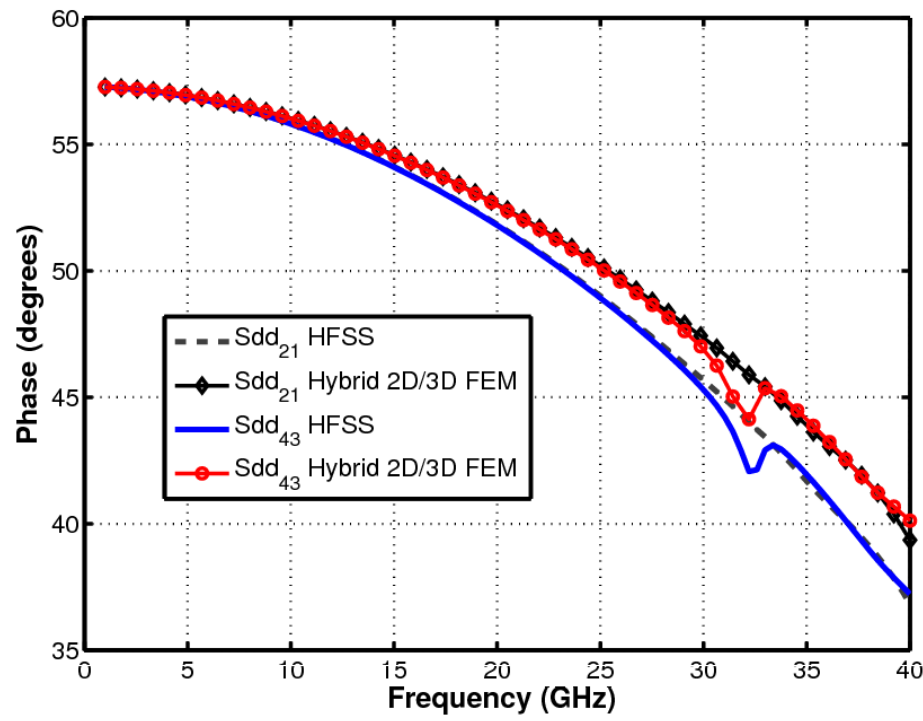
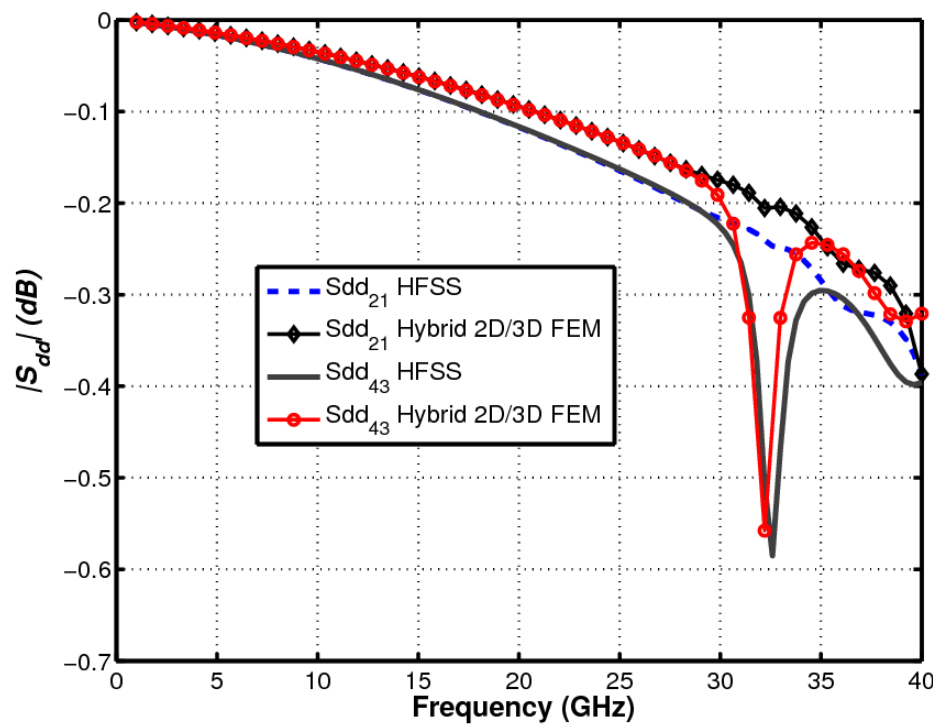
# Single-ended insertion loss



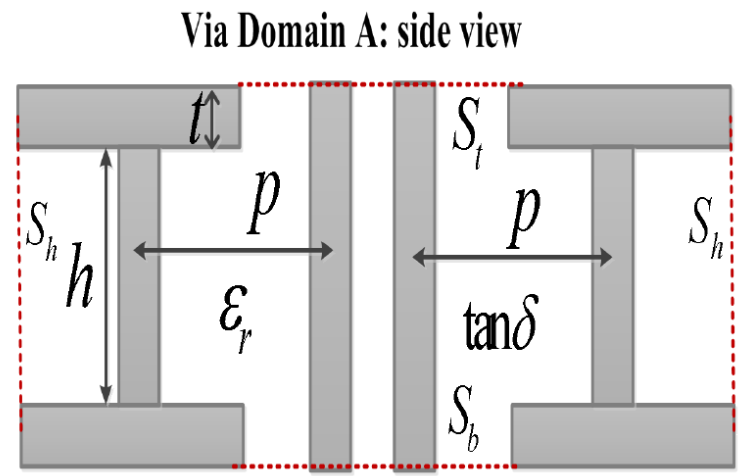
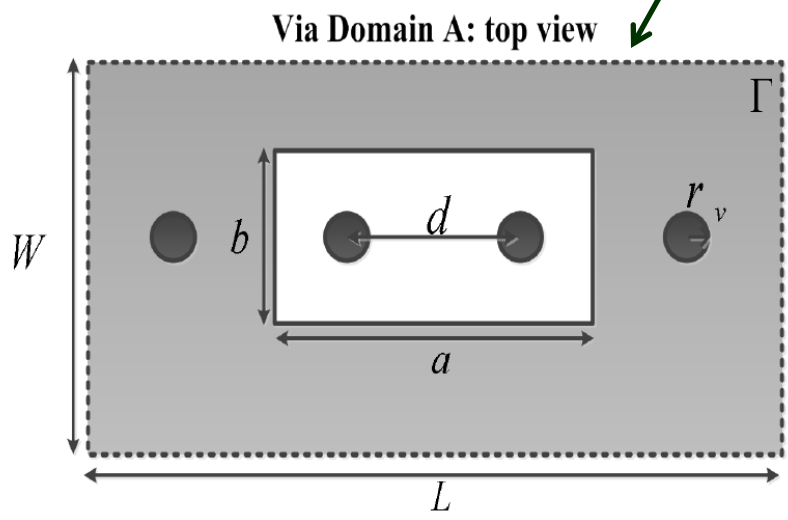
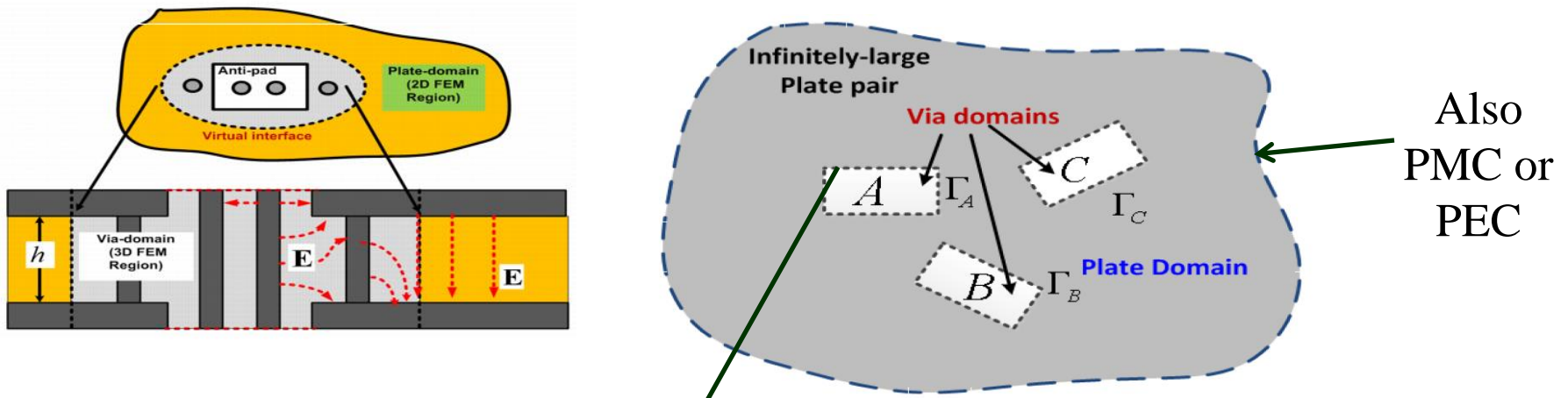
Again, the hybrid method is validated.



# Differential mode insertion loss

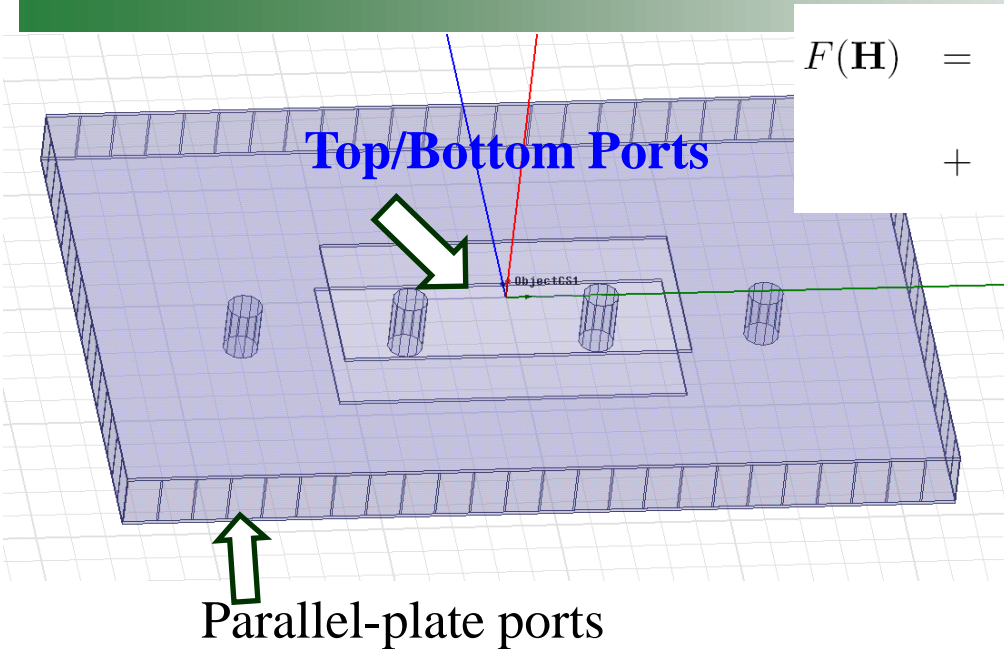


# Hybrid FEM and boundary integral equations



Via-domains: 3D FEM + Plate-domain: 2D BIE

# 3D FEM analysis for via-domains



$$F(\mathbf{H}) = \frac{1}{2} \int \int \int_V (\nabla \times \mathbf{H} \cdot \epsilon_r^{-1} \nabla \times \mathbf{H} - k_0^2 \mathbf{H} \cdot \mathbf{H}) dV + j\omega\epsilon_0 \int \int_S \mathbf{n} \times \mathbf{H} \cdot \mathbf{E}_s dS \quad (1)$$

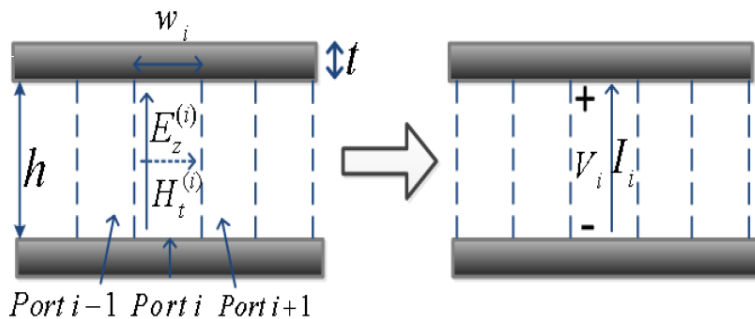
$$\mathbf{n} \times \mathbf{H} = J_z \mathbf{e}_z$$

$$\mathbf{E}_s = E_z \mathbf{e}_z$$

$$V_i = -E_z^{(i)} h$$

$$I_i = w_i J_z^{(i)}$$

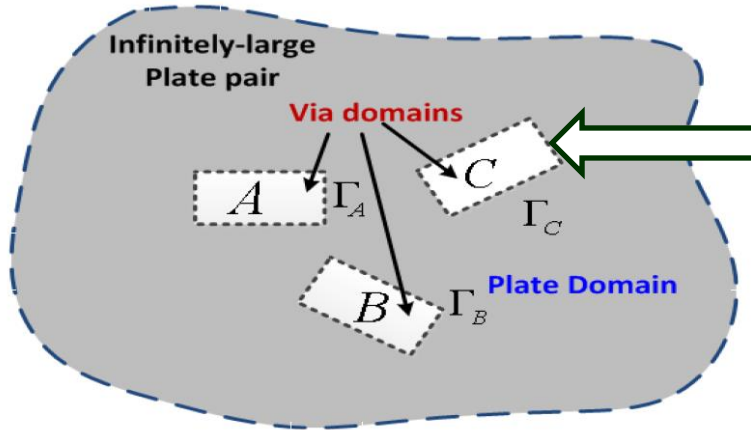
## S-parameter for a via domain



$$\begin{bmatrix} \mathbf{b}_t^v \\ \mathbf{b}_b^v \\ \mathbf{b}_h^v \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{tt}^v & \mathbf{S}_{tb}^v & \mathbf{S}_{th}^v \\ \mathbf{S}_{bt}^v & \mathbf{S}_{bb}^v & \mathbf{S}_{bh}^v \\ \mathbf{S}_{ht}^v & \mathbf{S}_{hb}^v & \mathbf{S}_{hh}^v \end{bmatrix} \begin{bmatrix} \mathbf{a}_t^v \\ \mathbf{a}_b^v \\ \mathbf{a}_h^v \end{bmatrix}$$



# Boundary integral equation for plate-domains



Parallel-plate ports

$$\begin{aligned} V_i &= -E_z^{(i)} h \\ I_i &= w_i J_z^{(i)} \end{aligned}$$

**Radiated E-fields by electric currents J**

$$E_z^J(\mathbf{r}) = -\frac{k\eta}{4} \int_{\Gamma} J_z(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\Gamma.$$

**Radiated E-fields by magnetic currents M (Ez here)**

$$E_z^M(\mathbf{r}) = \frac{j}{4} \int_{\Gamma} E_z(\mathbf{r}') \nabla \left[ H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) \right] \cdot \mathbf{n}' d\Gamma,$$

**Boundary integral equations on virtual interfaces and plate edges**

$$E_z(\mathbf{r}) = E_z^M(\mathbf{r}) + E_z^J(\mathbf{r})$$

$$\begin{aligned} & - \frac{k\eta}{4} \int_{\Gamma} J_z(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\Gamma \\ & = E_z(\mathbf{r}) - \frac{j}{4} \int_{\Gamma} E_z(\mathbf{r}') \nabla \left[ H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) \right] \cdot \mathbf{n}' d\Gamma \end{aligned}$$

# Plate-domain model: $Z_{pp}$

$$\begin{aligned}
 & - \frac{k\eta}{4} \int_{\Gamma} J_z(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\Gamma \\
 & = E_z(\mathbf{r}) - \frac{j}{4} \int_{\Gamma} E_z(\mathbf{r}') \nabla \left[ H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) \right] \cdot \mathbf{n}' d\Gamma
 \end{aligned}$$

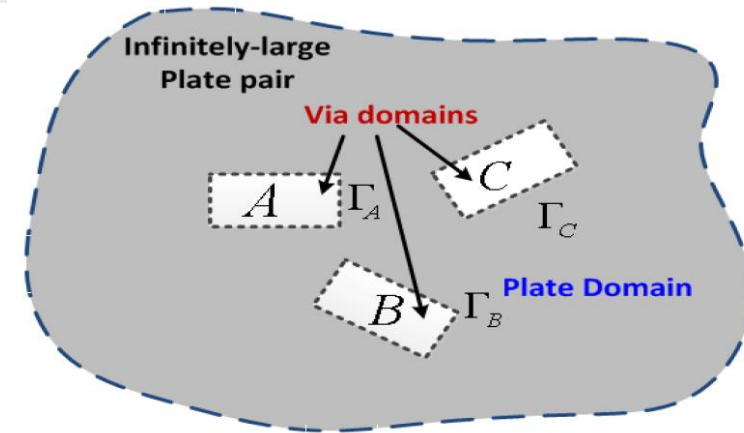
$$V_i = -E_z^{(i)} h$$

$$I_i = w_i J_z^{(i)}$$

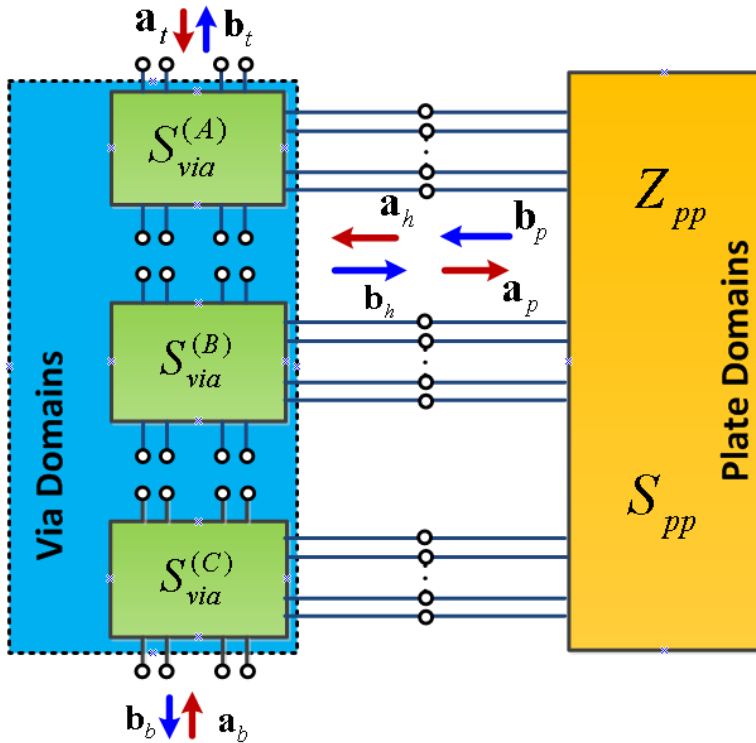
**Integral equation solved by the method of moments**

$$\mathbf{V}_p = \mathbf{Z}_{pp} \mathbf{I}_p$$

$Z_{pp}$  related tangential H-fields and E-fields on the virtual interfaces between 3D FEM via domains and the plate domain.



# Integrated via domains and plate domain



## Combined S-parameters of via domains

$$\begin{bmatrix} b_t \\ b_b \\ b_h \end{bmatrix} = \begin{bmatrix} S_{tt} & S_{tb} & S_{th} \\ S_{bt} & S_{bb} & S_{bh} \\ S_{ht} & S_{hb} & S_{hh} \end{bmatrix} \begin{bmatrix} a_t \\ a_b \\ a_h \end{bmatrix}$$

## Connection of S-parameters of via domains and plat domain

$$\begin{bmatrix} b_t \\ b_b \end{bmatrix} = \begin{bmatrix} S_{tt}^{(l)} & S_{tb}^{(l)} \\ S_{bt}^{(l)} & S_{bb}^{(l)} \end{bmatrix} \begin{bmatrix} a_t \\ a_b \end{bmatrix}$$

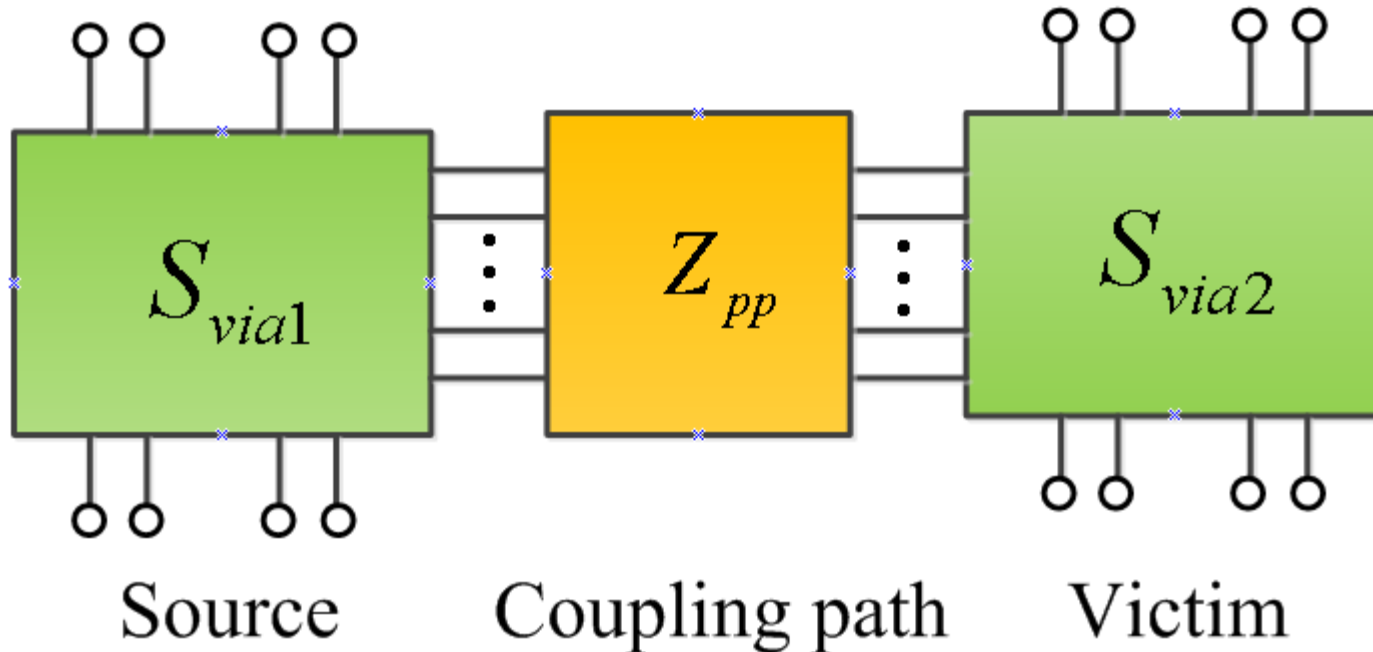
$$S_{tt}^{(l)} = S_{tt} + S_{th} S_{pp} (\mathbf{I} - S_{hh} S_{pp})^{-1} S_{ht}$$

$$S_{tb}^{(l)} = S_{tb} + S_{th} S_{pp} (\mathbf{I} - S_{hh} S_{pp})^{-1} S_{hb}$$

$$S_{bt}^{(l)} = S_{bt} + S_{bh} S_{pp} (\mathbf{I} - S_{hh} S_{pp})^{-1} S_{ht}$$

$$S_{bb}^{(l)} = S_{bb} + S_{bh} S_{pp} (\mathbf{I} - S_{hh} S_{pp})^{-1} S_{hb}$$

# Benefits of hybrid 3D FEM and BIE

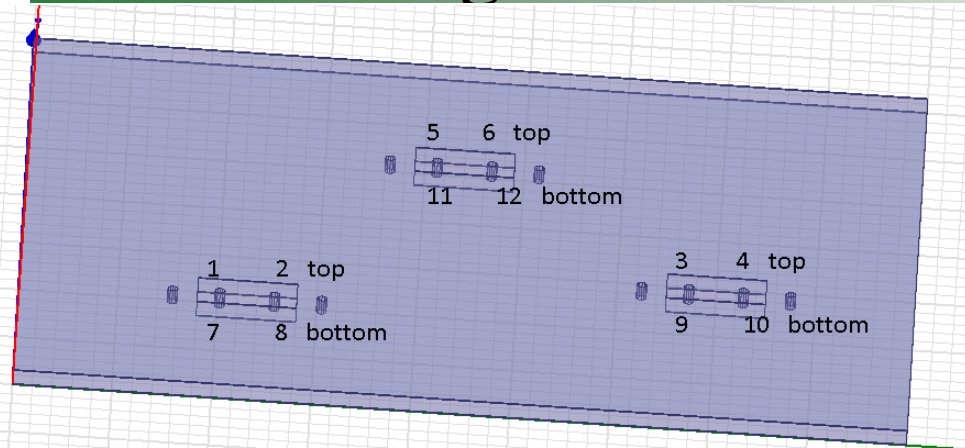


1. S-parameter of each via domain can be regarded as intrinsic properties of the via structure as it does not change with surrounding environment.
2. S-parameter of the plate domain characterizes the coupling path among via domains.

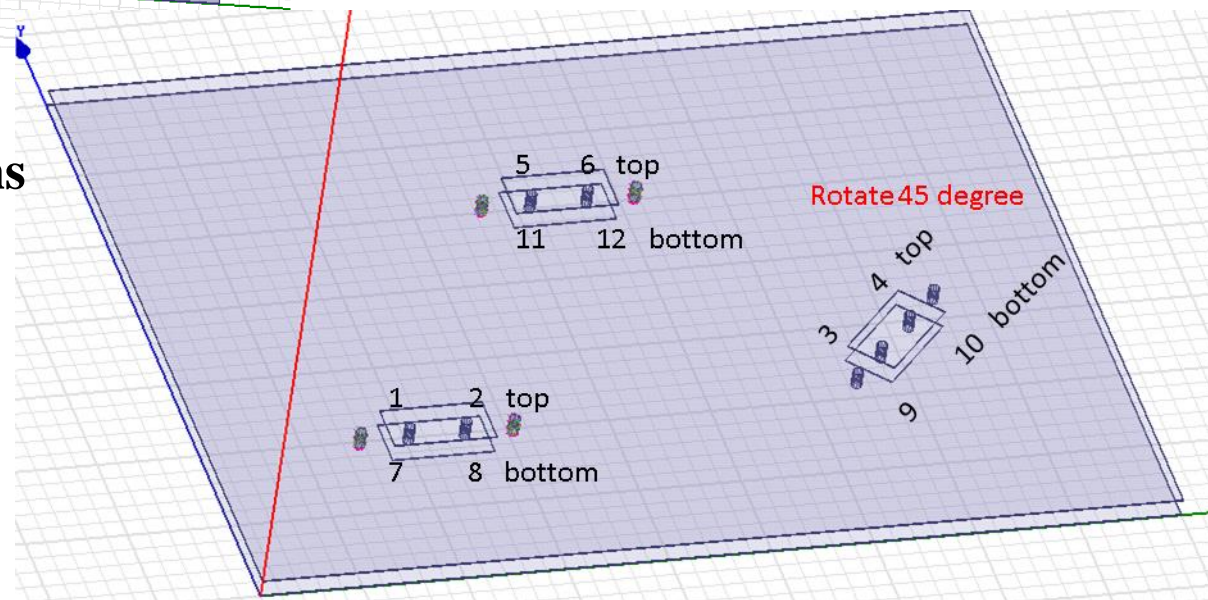
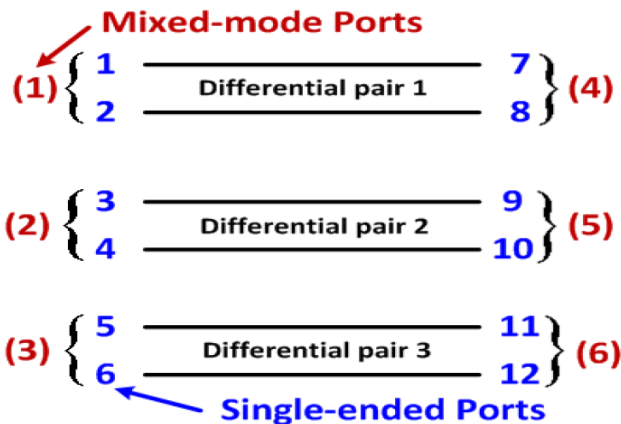
# Three via structures in a plate pair with PML and PMC edges

## Case 1: three parallel via domains

Radiation boundary condition and perfectly magnetic boundaries are used for 4 edges in HFSS simulation.

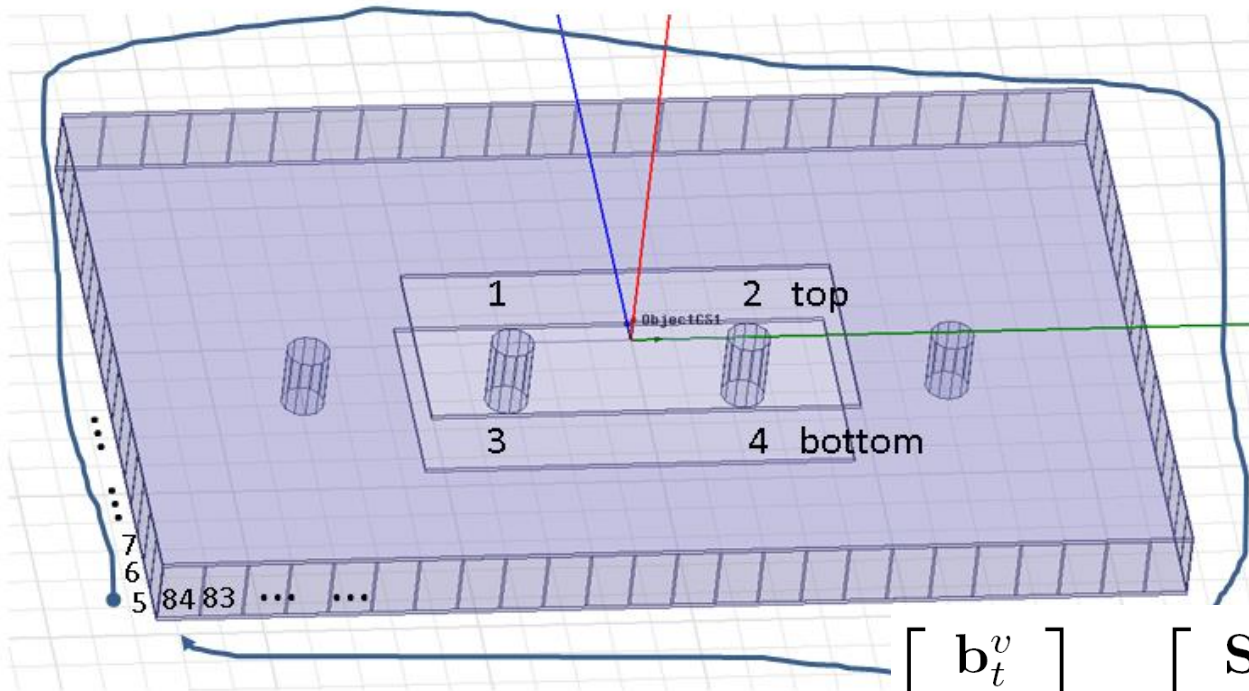


## Case 2: three via domains



# Hybrid 3D FEM and BIE: via domain

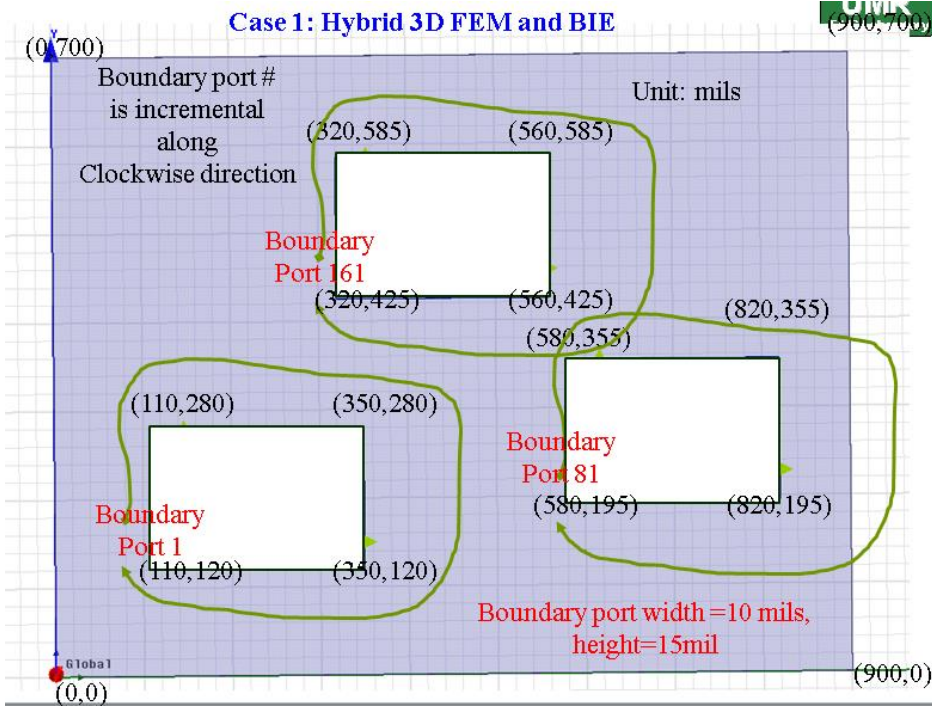
Port #



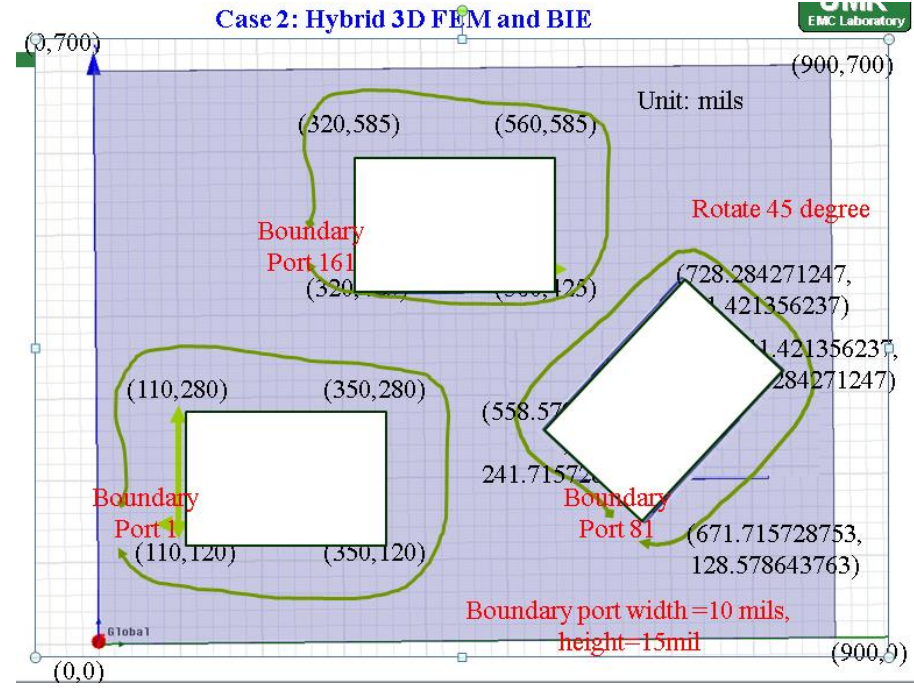
$$\begin{bmatrix} \mathbf{b}_t^v \\ \mathbf{b}_b^v \\ \mathbf{b}_h^v \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{tt}^v & \mathbf{S}_{tb}^v & \mathbf{S}_{th}^v \\ \mathbf{S}_{bt}^v & \mathbf{S}_{bb}^v & \mathbf{S}_{bh}^v \\ \mathbf{S}_{ht}^v & \mathbf{S}_{hb}^v & \mathbf{S}_{hh}^v \end{bmatrix} \begin{bmatrix} \mathbf{a}_t^v \\ \mathbf{a}_b^v \\ \mathbf{a}_h^v \end{bmatrix}$$

84-by-84 S-parameter matrix for a single via domain.

# Plate domain in hybrid 3D FEM and BIE

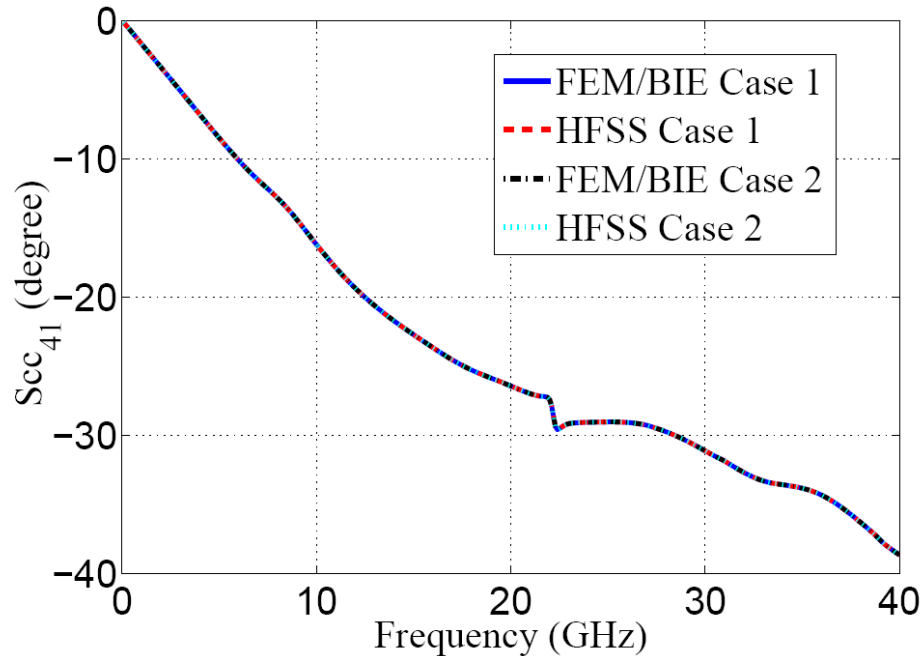
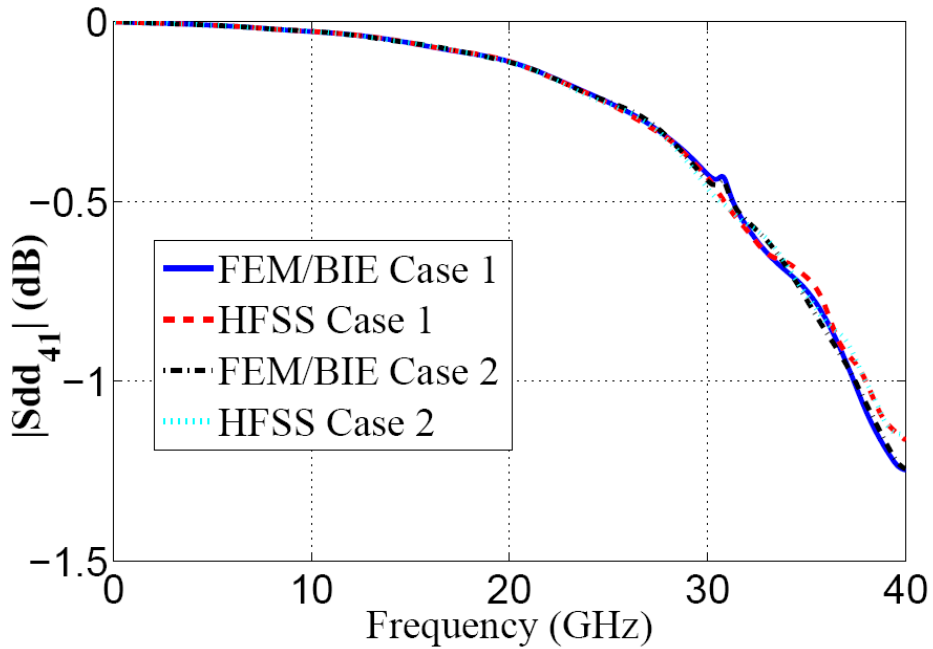


Case 1



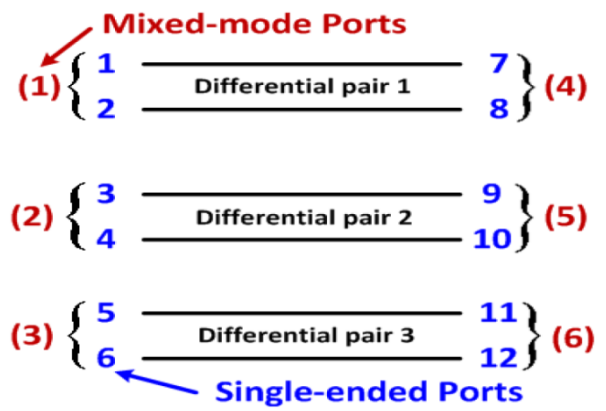
Case 2

# Insertion loss: infinitely-large plate pair



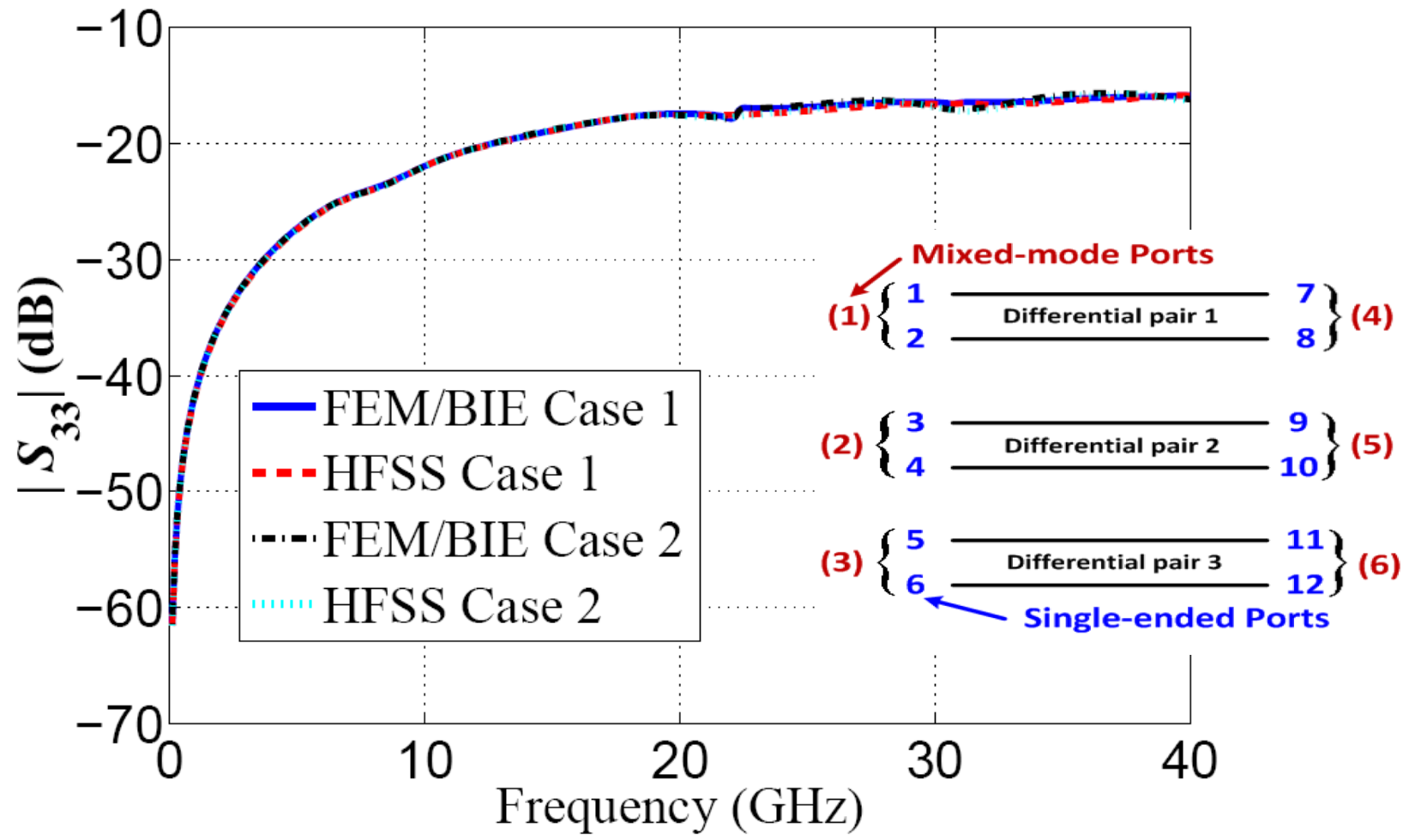
**Differential mode**

**Common mode**

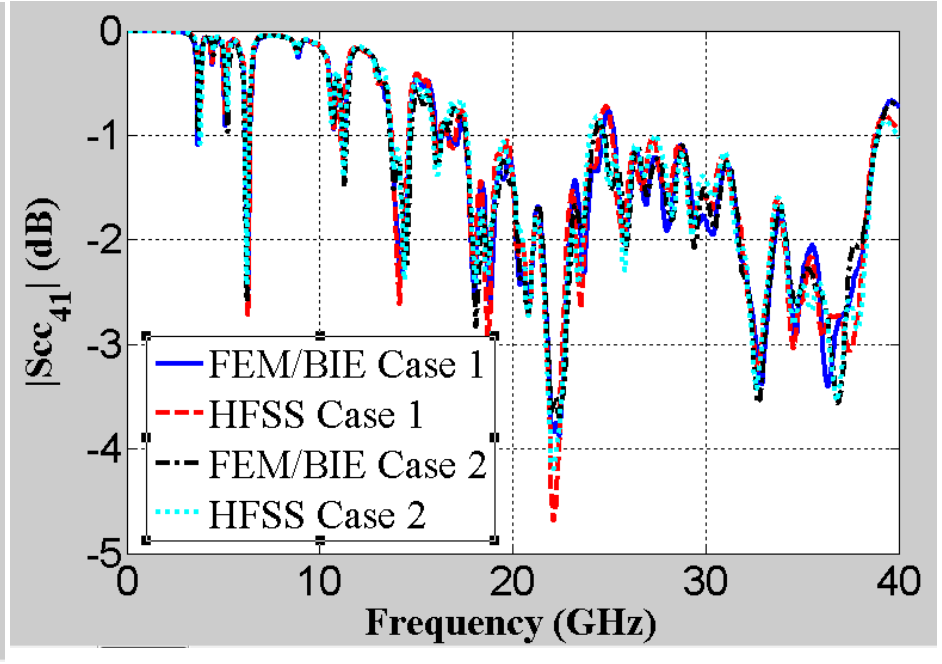
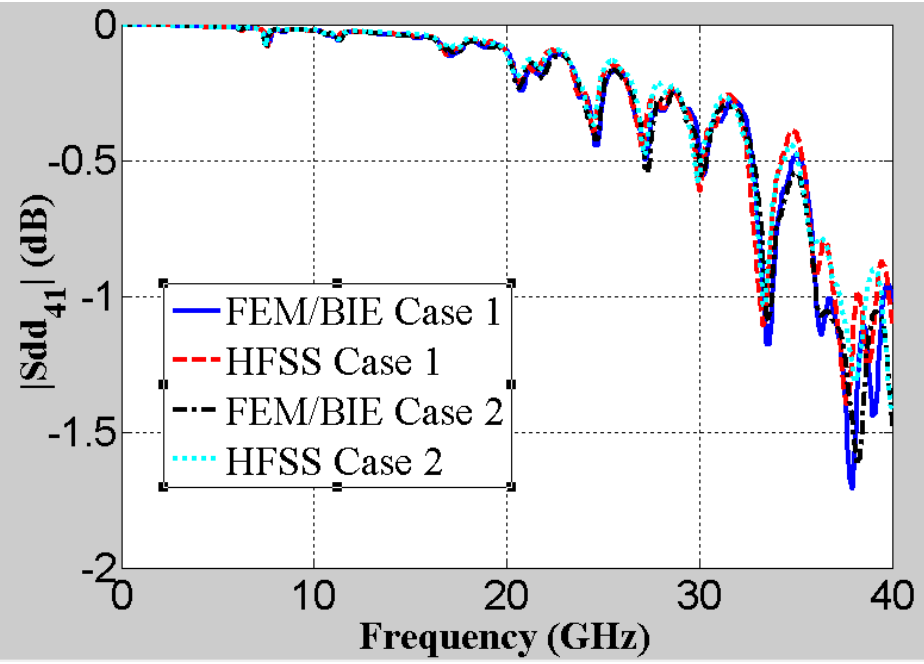




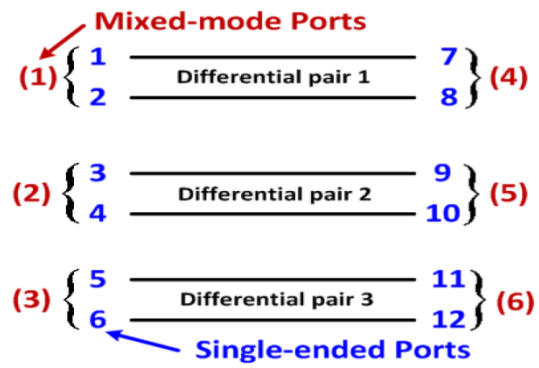
# Single-ended return loss



# Insertion loss: finite plate pair with PMC edges

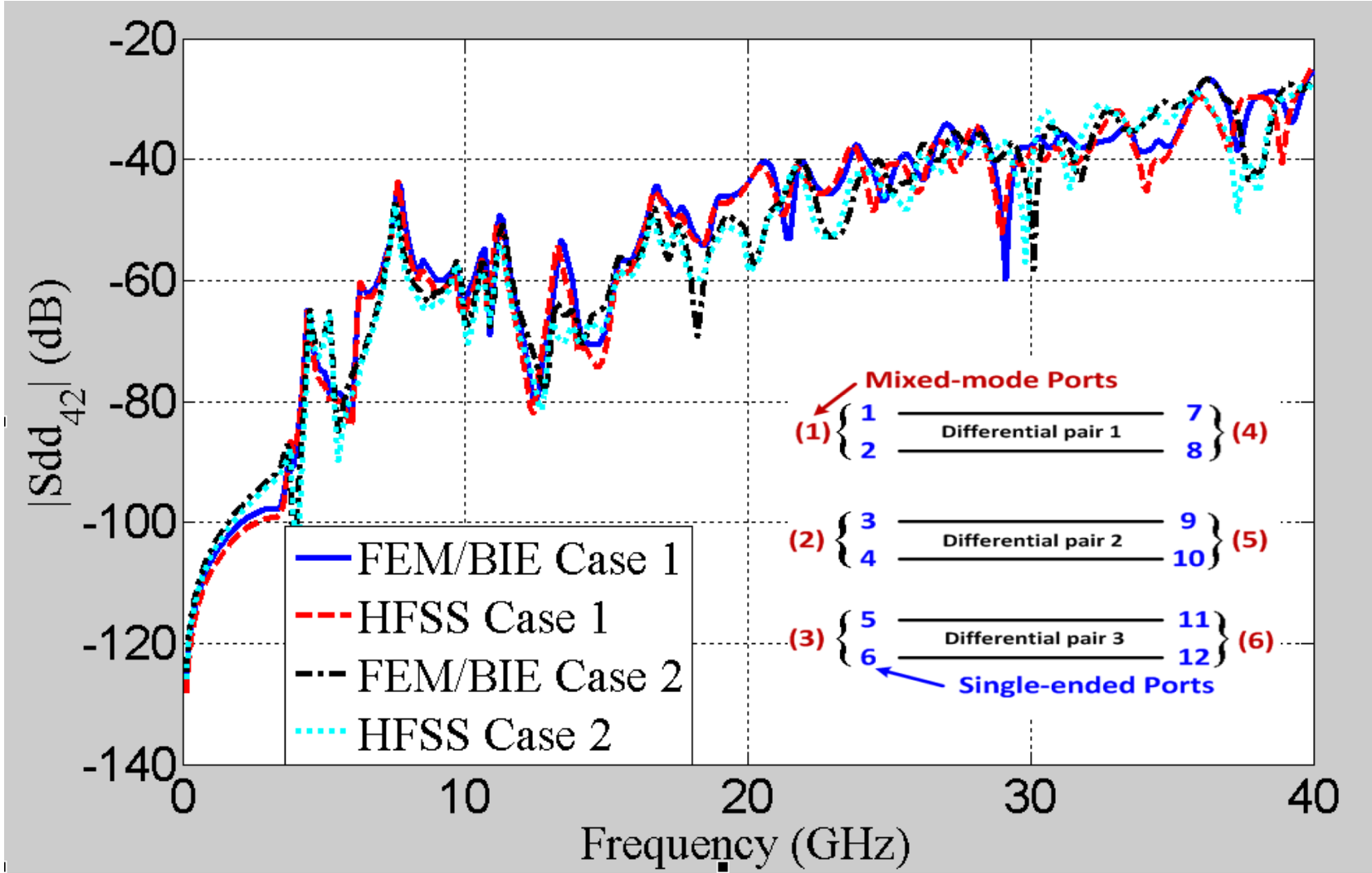


**Differential mode**

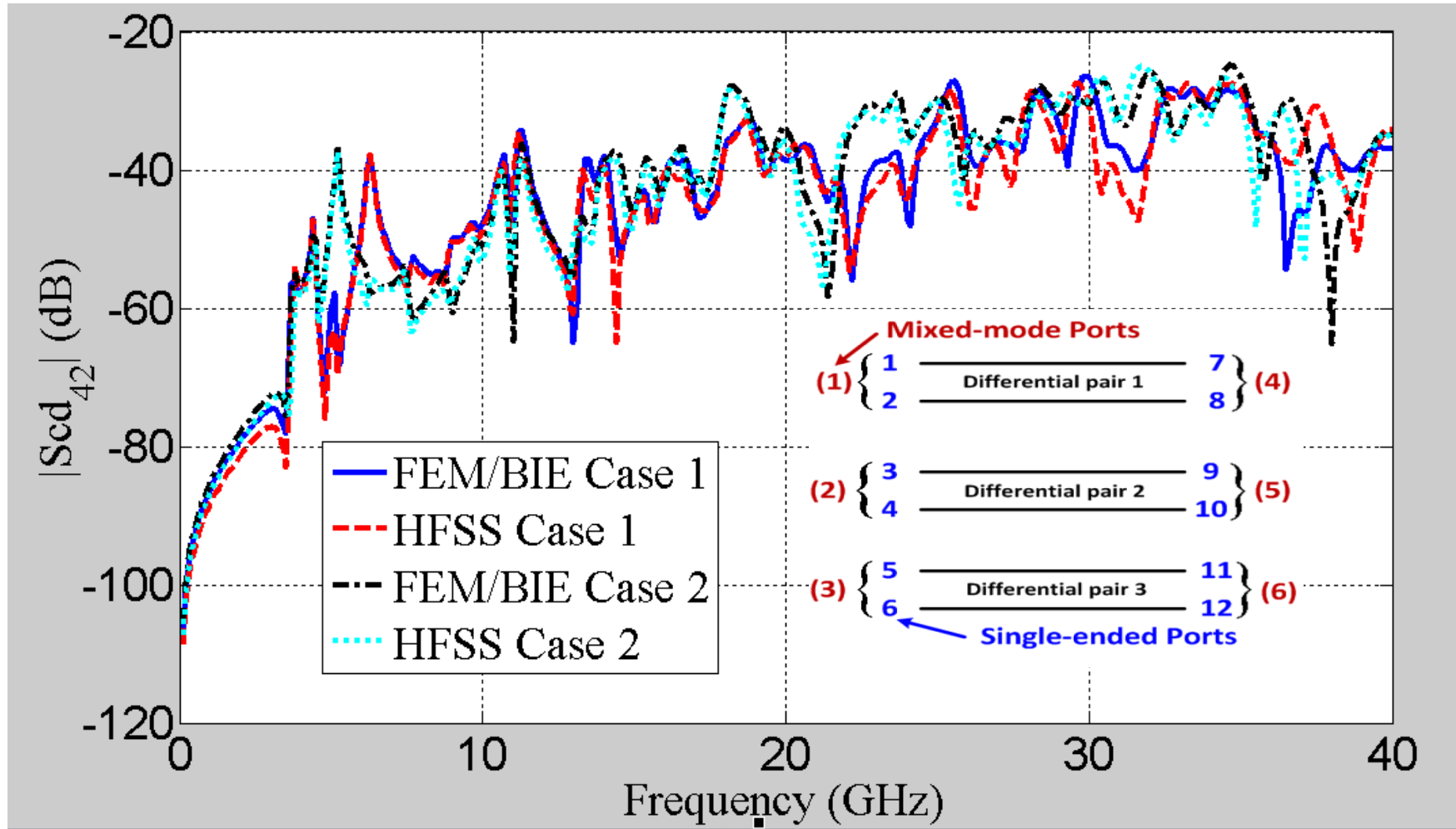


**Common mode**

# Forward Cross Talk

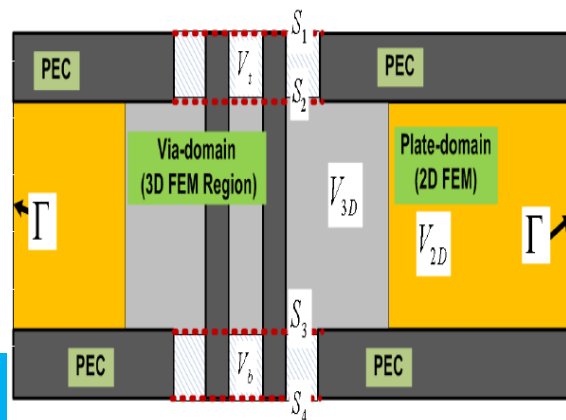


# Mixed-mode cross-talk



# Future work: higher-order modes in anti-pads

## Test TEM assumption in via holes on S1 and S2

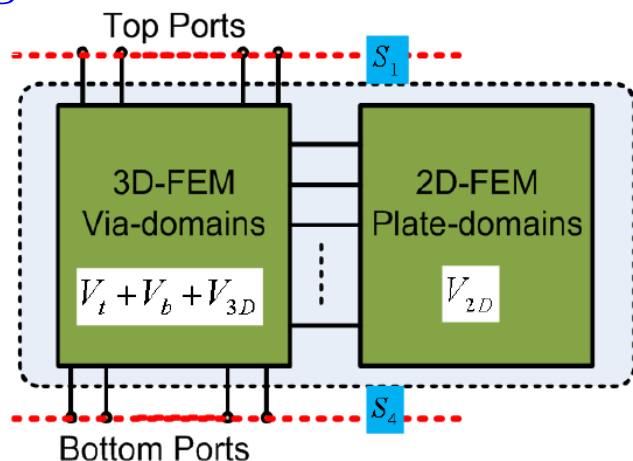


Hybrid 3D/2D FEM

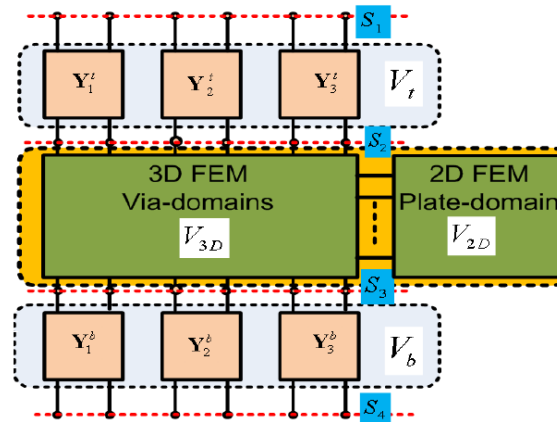
Combined admittance matrix (CAM)

Higher-order modes considered

Higher-order modes **not** considered



Comparison



If TEM mode assumptions on S2 and S3 are good enough, two approaches should get similar results.

# Comparisons: different plate thickness

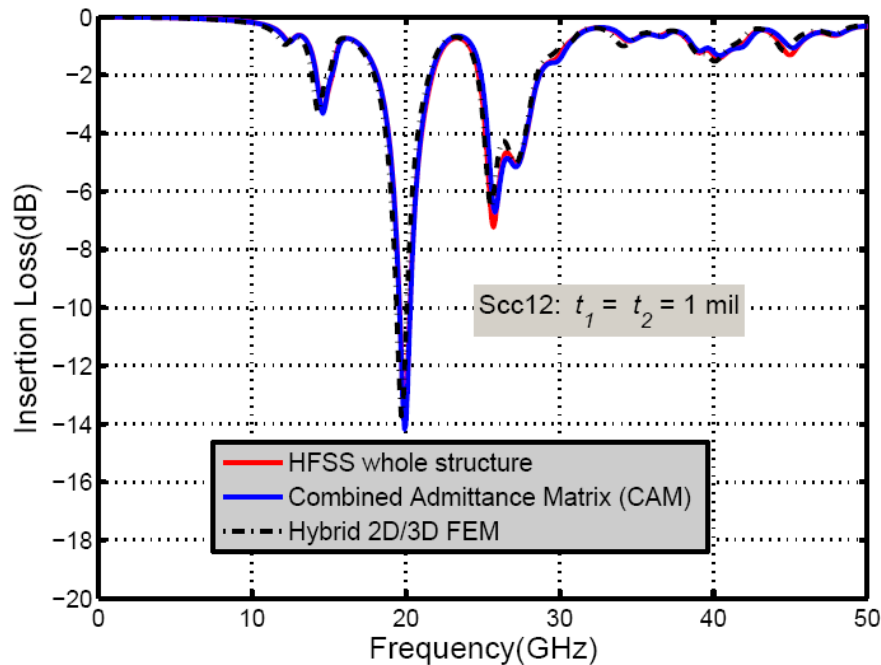


Plate thickness: 1 mils

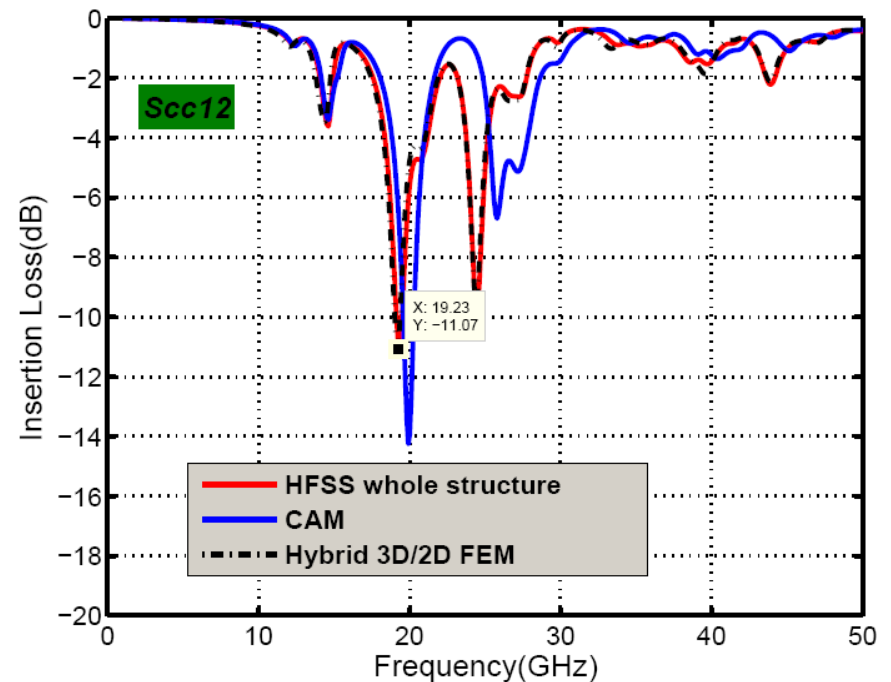
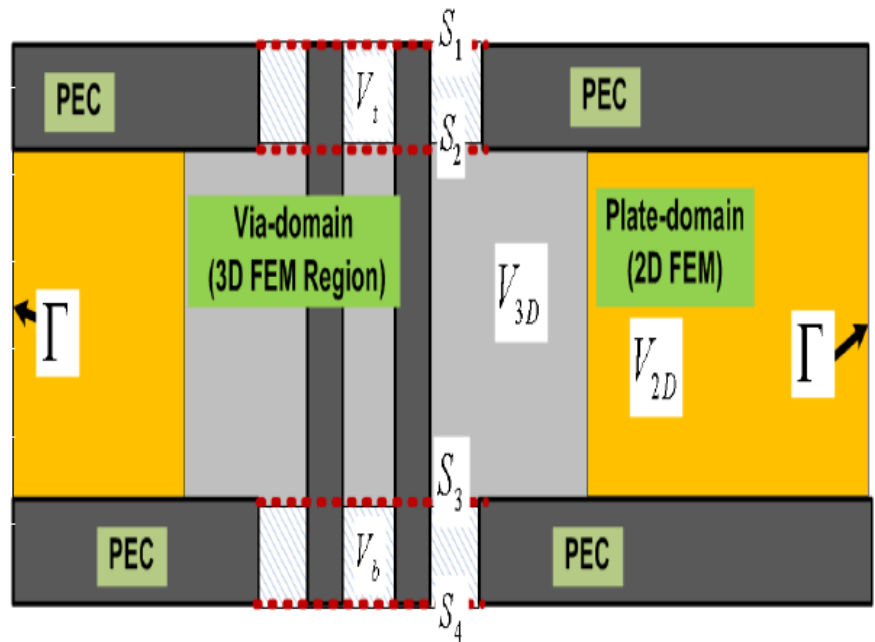


Plate thickness: 10 mils

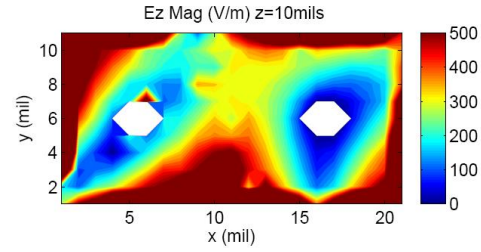
**Question:** why smaller plate thickness leads to smaller differences between two approaches?

# Explanation: plate thickness 10 mils

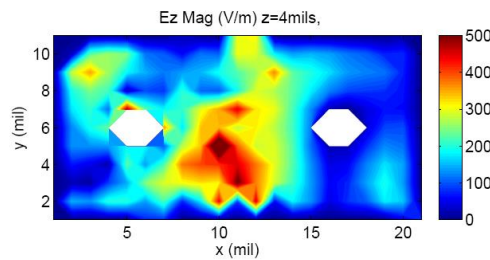


## TEM modes as sources on S1

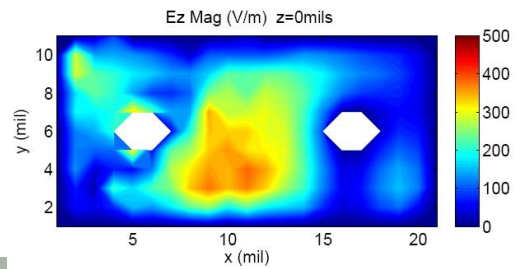
### Higher-order E-modes on S2



### Higher-order E-modes on a cross-section between S1 and S2



### Higher-order E-modes on S1



**Note:**

1. Strictly speaking, TEM assumption on S2 or S3 is not correct but perhaps an acceptable approximation.
2. More rigorous method will include the higher-order modes as they can couple from one layer of plate pair to another layer.

# Summary

- Critical roles of via modeling in SI/PI and EMI analysis
- Review of hybrid field-circuit models of vias for conventional via structures.
- Two recently developed hybrid methods, hybrid 3D/2D FEM and hybrid 3D FEM and 2D BIE, are introduced.

**Comments and Questions?**

**Suggestion for future work?**