Causality problems related to the numerical modeling of interconnects and connectors

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S-parameter models are widely used to model high speed interconnects and connectors

- Model accurately frequency dependent behavior (Skin effect, resonances, filter effects, …)
- Easy to generate: NWA measurements, Full wave simulators
- Accurate simulations require high-quality S-parameters
  - Passive
  - Reciprocal
  - Causal
  - …
- Practical considerations
  - Bandwidth limited
  - Tabulated: only known at discrete values
Causality is a simple, intuitive concept.
- Cause and effect
- There can be no response of a system before the system is excited.
Causality for continuous signals with an infinite bandwidth
Causality for continuous signals with infinite bandwidth

Time domain condition for causality

\[ v(t) = 0 \quad t < 0 \]

or

\[ v_{\text{even}}(t) = \text{sign}(t) \cdot v_{\text{odd}}(t) \]

\[ v_{\text{odd}}(t) = \text{sign}(t) \cdot v_{\text{even}}(t) \]

\[ \text{sign}(t) = \begin{cases} 
-1 & t < 0 \\
0 & t = 0 \\
1 & t > 0 
\end{cases} \]
Time domain condition for causality

\[ v(t) \neq 0 \quad t < 0 \]

or

\[ v_{\text{even}}(t) \neq \text{sign}(t) \cdot v_{\text{odd}}(t) \]

\[ v_{\text{odd}}(t) \neq \text{sign}(t) \cdot v_{\text{even}}(t) \]

v(t) is not causal

\[ \text{sign}(t) = \begin{cases} 
-1 & t < 0 \\
0 & t = 0 \\
1 & t > 0 
\end{cases} \]
**Causality for continuous signals with infinite bandwidth**

**Time domain condition for causality**

\[ v(t) = 0 \quad t < 0 \]

or

\[ v_{\text{even}}(t) = \text{sign}(t) \cdot v_{\text{odd}}(t) \]
\[ v_{\text{odd}}(t) = \text{sign}(t) \cdot v_{\text{even}}(t) \]

with \( \text{FFT}[\text{sign}(t)] = \frac{2}{j2\pi f} \)

**Frequency domain condition for causality**

\[ V_R(f) = \frac{1}{\pi} \text{P} \int_{-\infty}^{+\infty} \frac{V_1(f')}{f - f'} \, df' \]
\[ V_1(f) = -\frac{1}{\pi} \text{P} \int_{-\infty}^{+\infty} \frac{V_R(f')}{f - f'} \, df' \]

**Fourier Transform**

REAL and IMAGINARY PART of the FREQUENCY DOMAIN REPRESENTATION OF THE SIGNAL are LINKED through the HILBERT TRANSFORM.
Causality for continuous functions with infinite bandwidth

\[ v_{\text{causal}}(t) = v_{\text{even}}(t) + \text{sign}(t) \cdot v_{\text{even}}(t) \]
\[ v_{\text{causal}}(t) = v_{\text{odd}}(t) + \text{sign}(t) \cdot v_{\text{odd}}(t) \]

\[ V_{\text{causal}}(f) = V_R(f) + j \cdot \text{SIGN}(f) \cdot V_R(f) \]
\[ V_{\text{causal}}(f) = \text{SIGN}(f) \cdot V_I(f) + j \cdot V_I(f) \]

Non-causality is shifted to positive time!
Causality for continuous functions with infinite bandwidth

\[ v_{\text{causal}}(t) = \frac{v_{\text{causal, even}}(t) + v_{\text{causal, odd}}(t)}{2} \]

\[ v_{\text{causal}}(t) = u(t) \cdot v(t) \quad \text{(Blind causality enforcement)} \]
Causality for discrete signals with a limited bandwidth
Causality for discrete bandwidth limited signals

Continuous, infinite bandwidth

Discrete bandwidth limited

- For discrete signals: time domain response = periodic response limited to one period $T_0$
- To avoid time domain leakage duration impulse response $(T_{max}) < T_0$
How to check for non-causalities

Causal or non-causal?

Non-causal

Causal
Causality for discrete bandwidth limited signals

Time domain condition for causality

\[ v(t_k) = 0 \quad -\frac{T_0}{2} < t_k < 0 \]

- To avoid time domain leakage, duration time domain response \( T_{\text{max}} < T_0 \)
- To be able to check causality, duration domain response \( T_{\text{max}} < \frac{T_0}{2} \)
• Time domain: Nyquist’s sampling theorem:

\[ \Delta t = \frac{1}{F_0} < \frac{1}{2F_{\text{max}}} \]

with \( F_{\text{max}} \) = the bandwidth of the sampled function
\[ \Delta f = \frac{1}{T_0} < \frac{1}{2T_{\text{max}}} \]

with \( T_{\text{max}} \) = the frequency duration of the sampled signal

\[ \Delta t = \frac{1}{F_0} \]

\[ \Delta f = \frac{1}{T_0} \]
Causality for discrete bandwidth limited signals

Continuous, infinite bandwidth

\[ v_{\text{even}}(t) = \text{sign}(t).v_{\text{odd}}(t) \]
\[ v_{\text{odd}}(t) = \text{sign}(t).v_{\text{even}}(t) \]

Discrete, bandwidth limited

\[ v_{s,\text{even}}(t_k) = \text{sign}_{T_0}(t_k).v_{s,\text{odd}}(t_k) \]
\[ v_{s,\text{odd}}(t_k) = \text{sign}_{T_0}(t_k).v_{s,\text{even}}(t_k) \]
Causality for discrete bandwidth limited signals

FREQUENCY DOMAIN CONDITION

Continuous, infinite bandwidth

\[ V_R(f) = \text{SIGN}(f) * V_I(f) \]
\[ V_I(f) = \text{SIGN}(f) * V_I(f) \]

Discrete, bandwidth limited

\[ V_R(f_k) = \text{SIGN}_{T_0}(f_k) \otimes V_I(f_k) \]
\[ V_I(f_k) = \text{SIGN}_{T_0}(f_k) \otimes V_R(f_k) \]

CAUSALITY ENFORCEMENT

\[ v_{\text{causal}}(t) = u(t) \cdot v(t) \]
\[ v_{s,\text{causal}}(t_k) = u_{T_0}(t_k) \cdot v_s(t_k) \]
Calculation of time domain responses
Calculation time domain response

Time domain convolution

\[ S(2,1) \ast x = \text{Pulse response} \]

Frequency domain multiplication

\[ S(2,1) \times \text{Spectrum pulse} = \text{Spectrum pulse response} \]
Causality issues due to bandwidth limitation
Time domain convolution

- Impulse response is non-causal: ringing
- Value every $\Delta t = \frac{1}{2F_{\text{max}}}$ = 25 ps

Excitation pulse * Impulse response

Length = 0.8”

$Z = 66 \text{ ohm}$

S(2,1)

Pulse response

Frequency [GHz]

IDFT

$0.2 \text{ pF} \quad 0.2 \text{ pF}$
Bandwidth limitation of an S-parameter model

Bandwidth unlimited signal

Bandwidth limiting filter (non-causal filter)

Bandwidth limited signal

\[ \text{IFT} \]

\[ \text{Gibbs phenomenon} \]
Frequency domain multiplication

Excitation pulse \( \times \) Pulse response → IDFT
Frequency domain multiplication vs. time domain convolution

Methods do not match perfectly!
Frequency domain multiplication vs. time domain convolution

Time domain convolution

Frequency domain multiplication

DFT

IDFT

Graphs showing the comparison between time domain convolution and frequency domain multiplication, including plots for pulse amplitude over time and frequency.
Frequency domain multiplication vs. time domain convolution

**Excitation pulse**

**Pulse Response**
Frequency domain multiplication vs. time domain convolution
Time domain sampling, when not done properly (Nyquist theorem) will introduce frequency domain leakage or aliasing.
Bandwidth limitation causes ringing and loss of resolution.
To reduce the ringing: need more bandwidth
To have more time points per bit: need more bandwidth

How much bandwidth is needed?

Minimum N points per bit required:

\[ F_{\text{max}} \geq \frac{N \cdot \text{Bitrate}}{2} \]

<table>
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<tr>
<th>Bitrate [Gb/s]</th>
<th>Bit time [ps]</th>
<th>N</th>
<th>( \Delta t ) [ps]</th>
<th>( F_{\text{max}} ) [GHz]</th>
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<td>4</td>
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<td>100</td>
<td>0.40</td>
<td>1250</td>
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</tbody>
</table>

Up to which frequency is actual data required?
When can I apply zero-padding?
Max bandwidth required

Notice: Main part of ringing in causal part of pulse response!
Max bandwidth required
Max bandwidth required

Amplitude non-causal part of ringing

Maximum model frequency [GHz]
Perfect channel defines upper limit for bandwidth.

Maximum non-causal amplitude $< 1e^{-3}$:
Maximum model frequency $= 200$ GHz for data rate $= 25$ Gb/s

Maximum non-causal amplitude $< 1e^{-4}$:
Maximum model frequency $> 300$ GHz for data rate $= 25.40$ Gb/s
Max bandwidth required

**Max bandwidth required**

**CHANNEL WITH LOSSES - Data rate = 25 Gb/s**

Maximum model frequency $F_{\text{max}}$ drops significantly if channel is lossy.
PERFECT CHANNEL, BANDWIDTH LIMITED BY BUTTERWORTH FILTER - Data rate = 25 Gb/s

Maximum model frequency $F_{\text{max}}$ drops further.
Techniques to minimize ringing

Discrete Fourier Transform vs Fourier Transform

IDFT

Inverse Fourier Transform
Techniques to minimize ringing - method 1

Bandwidth = 20 GHz

Bandwidth = 18.4 GHz
Techniques to minimize ringing - method 1

Impulse response

- Original + zero padding
- 18.4 GHz + zero padding

20 GHz
18.4 GHz
Techniques to minimize ringing - method 1

Pulse response
Techniques to minimize ringing - method 2

Bandwidth = 20 GHz

\[ S_{2,1}^{\text{new}}(f) = S_{2,1}(f)e^{-j\omega\tau} \]

\[ \tau = \alpha\Delta t \quad 0 \leq \alpha < 1 \]

Bandwidth = 20 GHz
Techniques to minimize ringing - method 2

- Original
- Original + small delay

- Original + zero padding
- Original + small delay + zero padding
Techniques to minimize ringing - method 2

- Original
- Original + small delay

Group delay at $F_{\text{max}} = n \Delta t$
Techniques to reduce ringing - increase resolution

Extrapolation to 100 GHz
Techniques to reduce ringing - increase resolution

Impulse response

\[ \Delta \text{Impulse response} = \text{(Infinite bandwidth} - \text{extrapolated signal)} \]
Techniques to reduce ringing - increase resolution

\[ \Delta \text{Pulse response} = \text{Infinite bandwidth} - \text{extrapolated signal} \]
Causality issues due to discretization
Frequency domain resolution

Z = 66 ohm
Length = 12”

S(2,1)

S(2,1) [dB]

Frequency [GHz]

Impulse response

Time [ns]

Impulse response

Time [ns]
Frequency domain resolution

Impulse response

[Graph showing impulse response with different frequency resolutions]

[Graph showing impulse response with different frequency resolutions]
**Frequency domain resolution**

- Continuous, infinite bandwidth
- Discrete, bandwidth limited

**Blind causality enforcement**

**Smart causality enforcement**
Frequency domain resolution

Continuous, infinite bandwidth

Discrete, bandwidth limited

Blind causality enforcement

Smart causality enforcement
Step response
Calculation step response

Time domain convolution

\[ S(2,1) * \text{Step} = \text{Step response} \]

Frequency domain multiplication

\[ \text{Spectrum step} \times S(2,1) = \text{Spectrum step response} \]
Calculation step response: example

S(1,1) Impulse response

- S(1,1) [dB]
- Frequency [GHz]
- Impulse response
- Time [ns]

- Infinite bandwidth
- 50 GHz
- 20 GHz
Calculation step response: example

S(1,1)

Impulse response
Calculation step response: example

S(1,1)

Step response

Step response

Step response

Step response
Calculation step response

Time domain convolution

Step - ramp

IDFT

Impulse response step - ramp

Step response
**Calculation step response**

\[
\text{step}(t) = \int_{-\infty}^{t} \text{impulse}(\tau) \, d\tau \\
\text{impulse}(t) = \frac{d\text{step}(t)}{dt}
\]

**Time domain convolution**

\[
\text{dStep/dt} \ast S(2,1) = \text{impulse response of the step response}
\]

\[
\text{Step response}
\]
Calculation step response

- step response
- dstep/dt

Impulse response of the step response

- step response
Impact on DC value when causality is enforced
**DC value and causality enforcement**

\[ DC = \int_{-\infty}^{+\infty} \text{impulse response}(t) \, dt \]

DC-value changes if causality is enforced and

\[ \int_{-\infty}^{0} \text{impulse response}(t) \, dt \neq 0 \]
Measurement of connector and footprint: impedance is not-causal
Step 1) Reduce ringing: make signal continuous at $F_0$:
Step 2) Enforce causality
DC value and causality enforcement

Impulse response

Step response

DC-value does not change if causality is enforced
DC value and causality enforcement