Causality problems related to the numerical modeling of interconnects and connectors

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Introduction

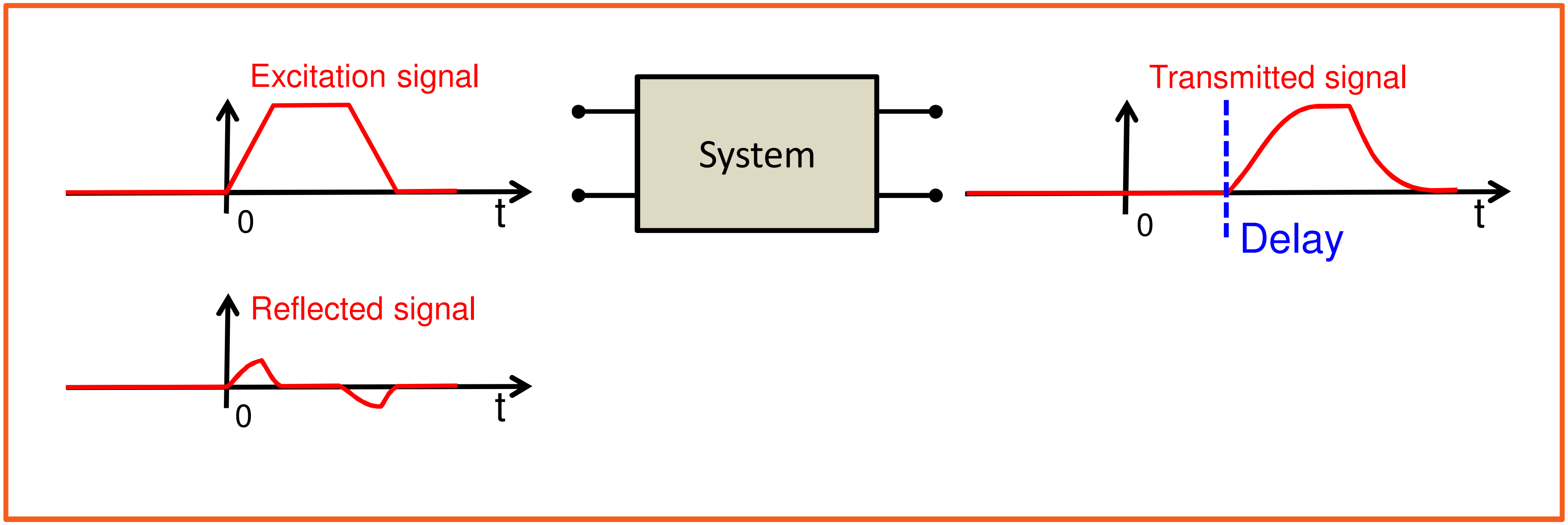
S-parameter models are widely used to model high speed interconnects and connectors

- Model accurately frequency dependent behavior (Skin effect, resonances, filter effects, …)
- Easy to generate: NWA measurements, Full wave simulators
- Accurate simulations require high-quality S-parameters
  - Passive
  - Reciprocal
  - Causal
  - …
- Practical considerations
  - Bandwidth limited
  - Tabulated: only known at discrete values

Causality

Causality is a simple, intuitive concept.

- Cause and effect
- There can be no response of a system before the system is excited.



Causality for continuous signals with an infinite bandwidth

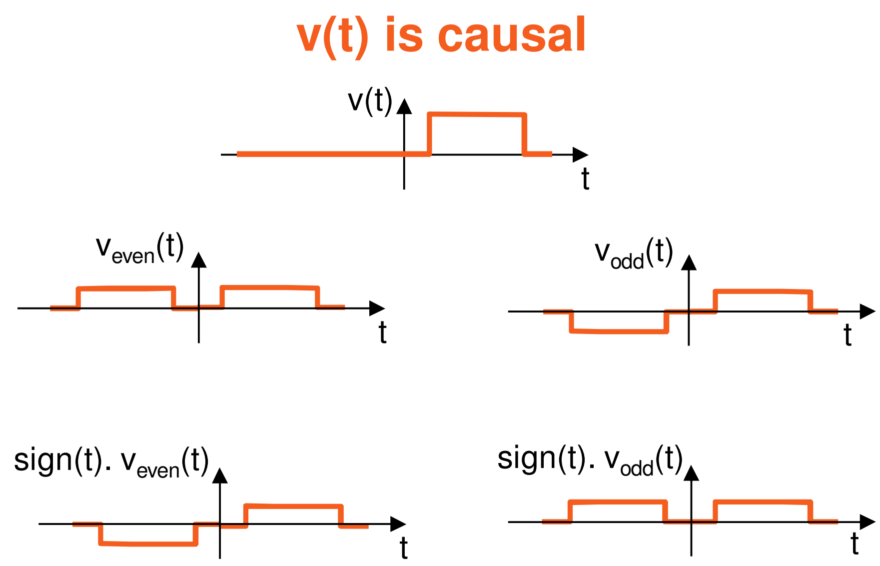
Causality for continuous signals with infinite bandwidth

Time domain condition for causality

$$v(t) = 0 \qquad t<0$$

or

$$v_{even}(t) = sign(t).v_{odd}(t)$$
$$v_{odd}(t) = sign(t).v_{even}(t)$$



$$sign(t) = \begin{cases} -1 & t<0 \\ 0 & t=0 \\ 1 & t>0 \end{cases}$$

Causality for continuous signals with infinite bandwidth

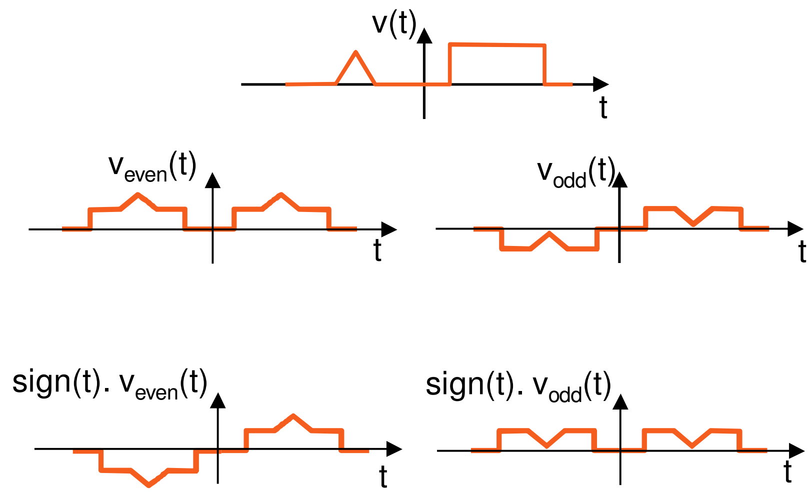
Time domain condition for causality

$$v(t) \neq 0 \qquad t<0$$

or

$$v_{even}(t) \neq sign(t).v_{odd}(t)$$
$$v_{odd}(t) \neq sign(t).v_{even}(t)$$

v(t) is not causal



$$sign(t) = \begin{cases} -1 & t<0 \\ 0 & t=0 \\ 1 & t>0 \end{cases}$$

Causality for continuous signals with infinite bandwidth

Time domain condition for causality

$$v(t) = 0 \qquad t<0$$

or

$$v_{even}(t) = sign(t).v_{odd}(t)$$
$$v_{odd}(t) = sign(t).v_{even}(t)$$

with $\quad FFT[sign(t)] = \frac{2}{j2\pi f}$

Frequency domain condition for causality

$$V_R(f) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{V_I(f')}{f - f'} df'$$

$$V_I(f) = -\frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{V_R(f')}{f - f'} df'$$

⇔

Fourier Transform

REAL and IMAGNINARY PART of the FREQUENCY DOMAIN REPRESENTATION OF THE SIGNAL are LINKED through the HILBERT TRANSFORM

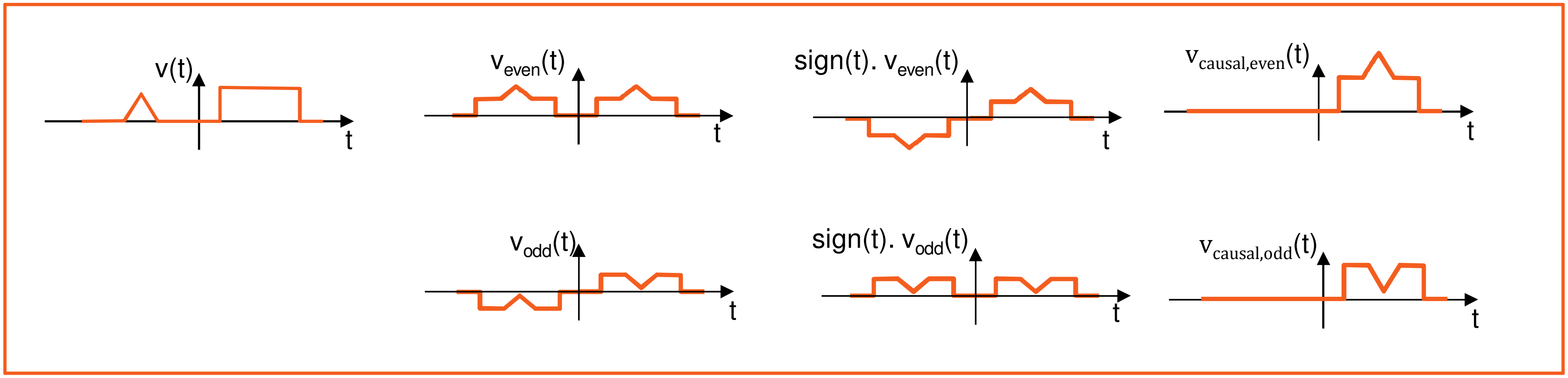
Causality for continuous functions with infinite bandwidth

$$v_{causal}(t) = v_{even}(t) + sign(t).\, v_{even}(t)$$

$$v_{causal}(t) = v_{odd}(t) + sign(t).\, v_{odd}(t)$$

$$V_{causal}(f) = V_R(f) + j.SIGN(f)^*V_R(f)$$

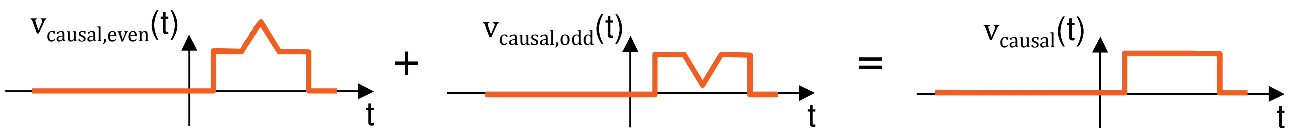
$$V_{causal}(f) = SIGN(f)^*V_I(f) + j.V_I(f)$$



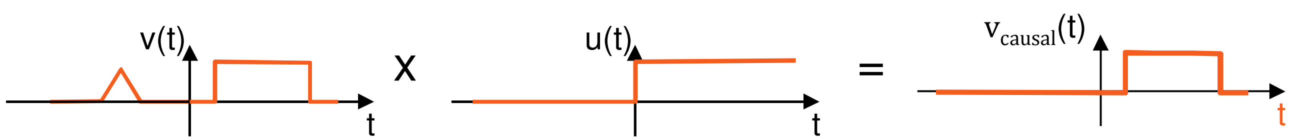
Non-causality is shifted to positive time!

Causality for continuous functions with infinite bandwidth

$$v_{causal}(t) = \frac{v_{causal,even}(t) + v_{causal,odd}(t)}{2}$$



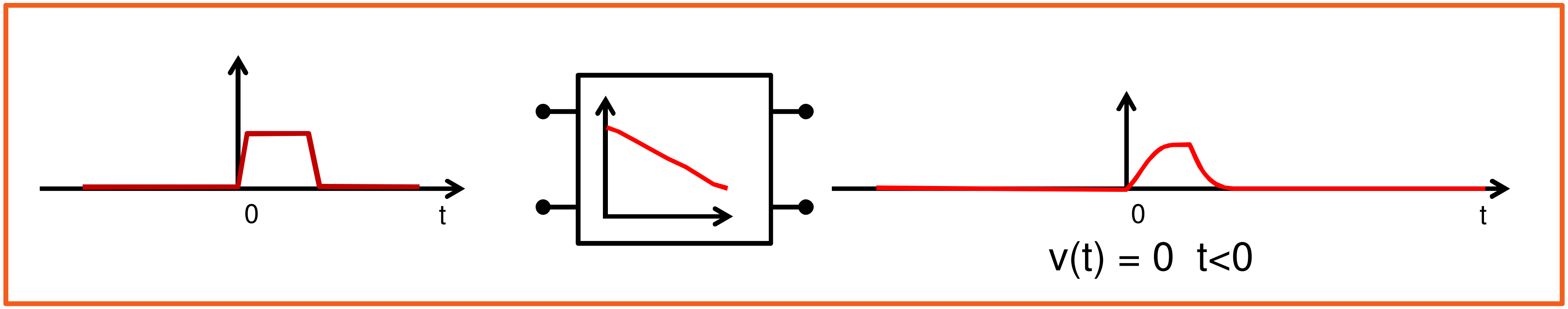
$v_{causal}(t) = u(t).v(t)$ (Blind causality enforcement)



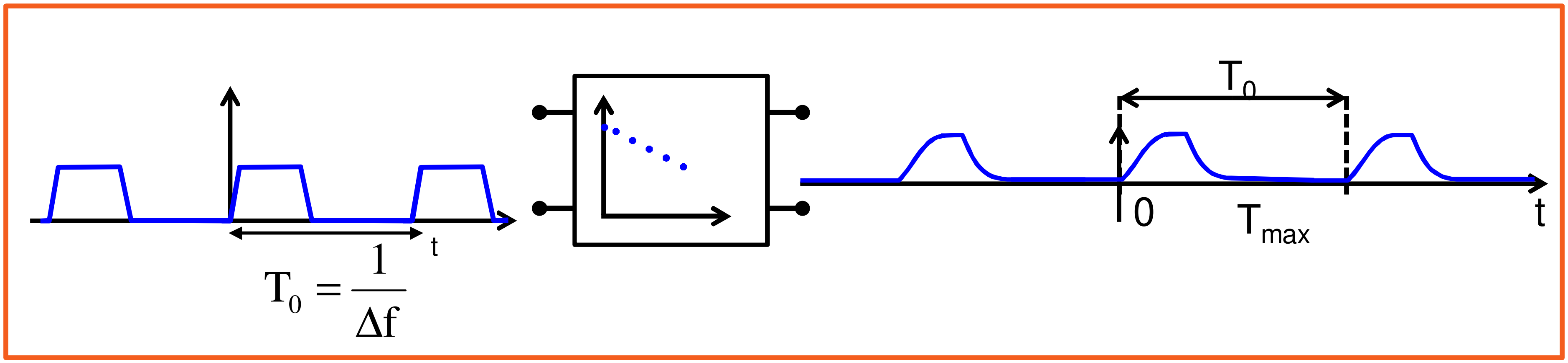
Causality for discrete signals with a limited bandwidth

Causality for discrete bandwidth limited signals

Continuous, infinite bandwidth

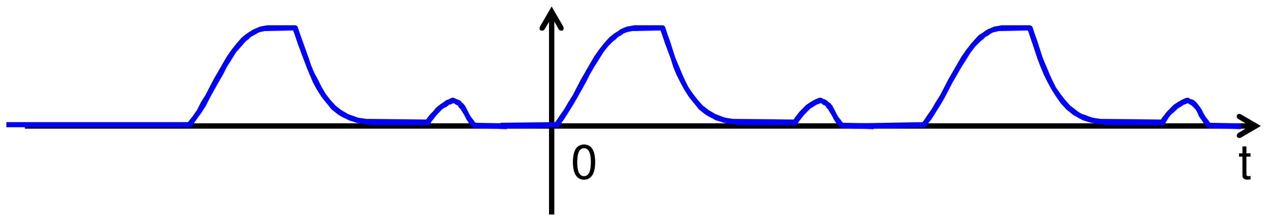


Discrete bandwidth limited

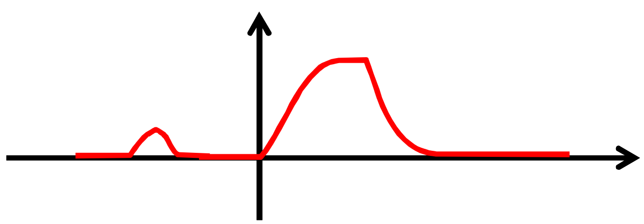


- For discrete signals: time domain response = periodic response limited to one period T_0
- To avoid time domain leakage duration impulse response (T_{max}) < T_0

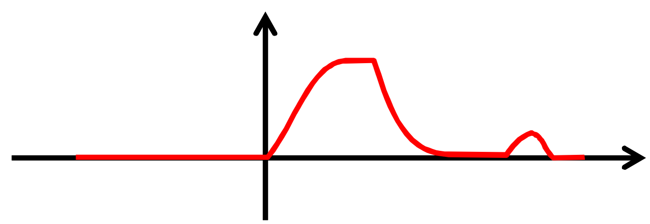
How to check for non-causalities



Causal or non-causal?

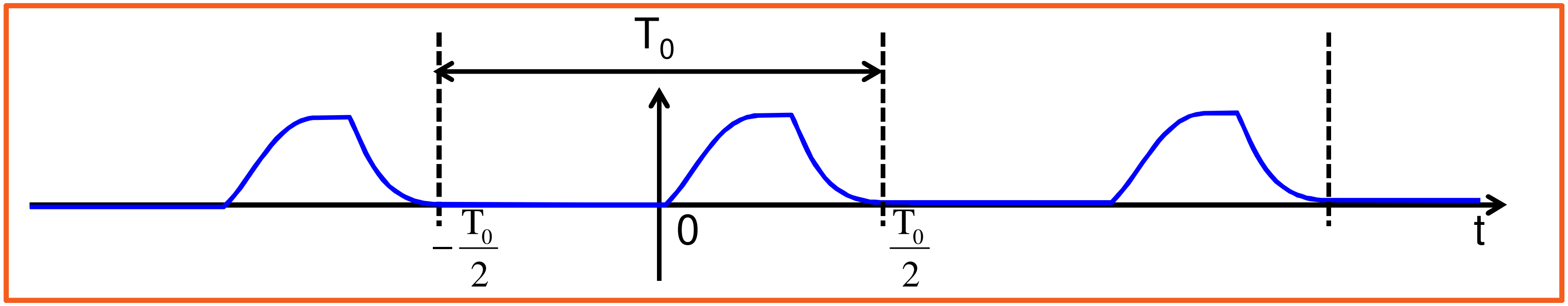


Non-causal



Causal

Causality for discrete bandwidth limited signals



Time domain condition for causality

$$v(t_k) = 0 \qquad -\frac{T_0}{2} < t_k < 0$$

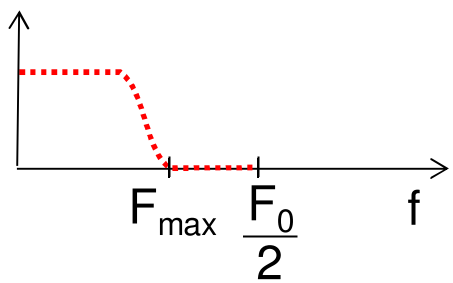
- To avoid time domain leakage, duration time domain response $T_{max} < T_0$
- To be able to check causality, duration domain response $T_{max} < \frac{T_0}{2}$

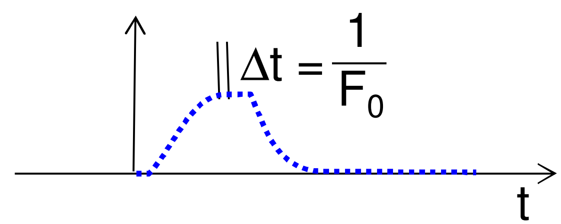
Similarity with Nuiquist's sampling theorem

- **Time domain: Nyquist's sampling theorem:**

$$\Delta t = \frac{1}{F_0} < \frac{1}{2.F_{max}}$$

with F_{max} = the bandwidth of the sampled function
= the frequency duration of the signal

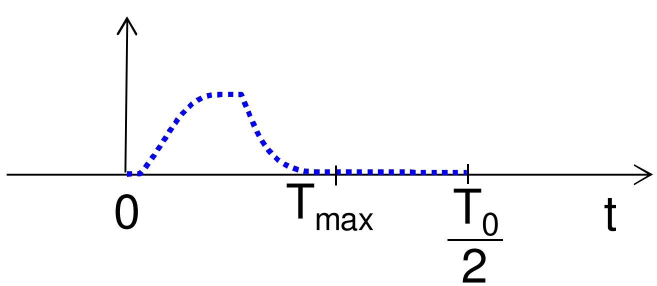


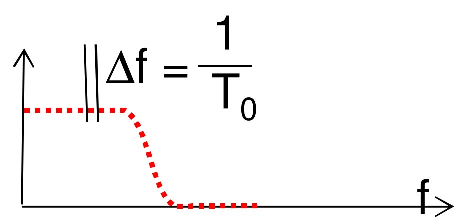


- **Frequency domain: dual of Nyquist sampling theorem:**

$$\Delta f = \frac{1}{T_0} < \frac{1}{2.T_{max}}$$

with T_{max} = the time duration of the sampled signal





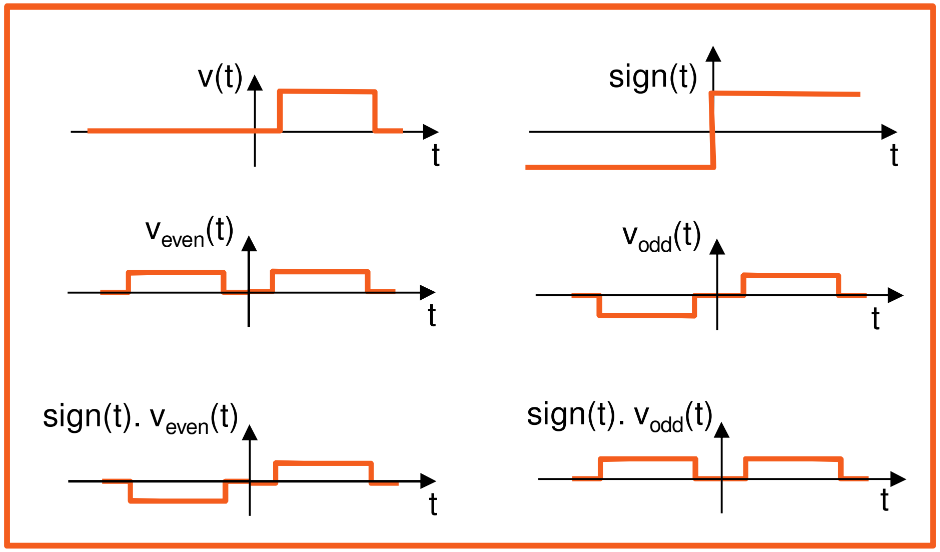
Causality for discrete bandwidth limited signals

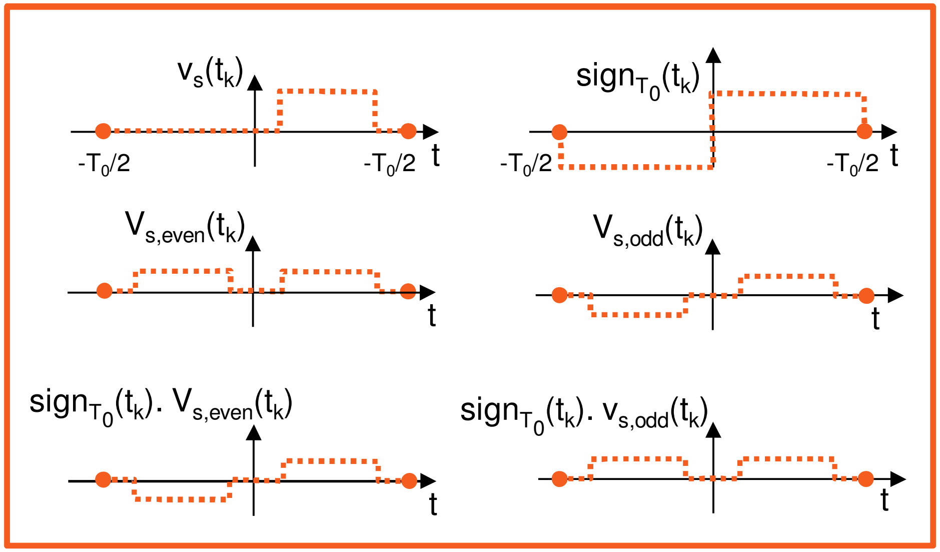
Continuous, infinite bandwidth

$$v_{even}(t) = sign(t).v_{odd}(t)$$
$$v_{odd}(t) = sign(t).v_{even}(t)$$

Discrete, bandwidth limited

$$v_{s,even}(t_k) = sign_{T_0}(t_k).v_{s,odd}(t_k)$$
$$v_{s,odd}(t_k) = sign_{T_0}(t_k).v_{s,even}(t_k)$$





Causality for discrete bandwidth limited signals

FREQUENCY DOMAIN CONDITION

Continuous, infinite bandwidth

$$V_R(f) = SIGN(f) * V_I(f)$$

$$V_I(f) = SIGN(f) * V_I(f)$$

Discrete, bandwidth limited

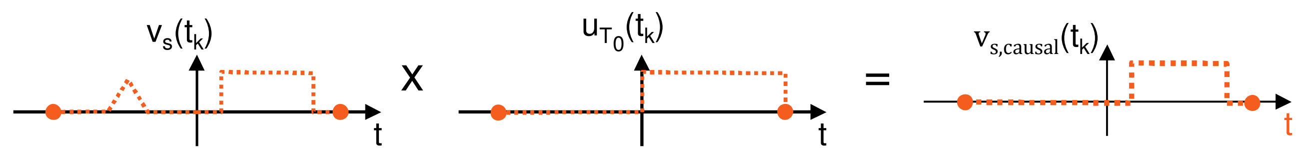
$$V_R(f_k) = SIGN_{T_0}(f_k) \circledast V_I(f_k)$$

$$V_I(f_k) = SIGN_{T_0}(f_k) \circledast V_R(f_k)$$

CAUSALITY ENFORCEMENT

$$v_{causal}(t) = u(t).v(t)$$

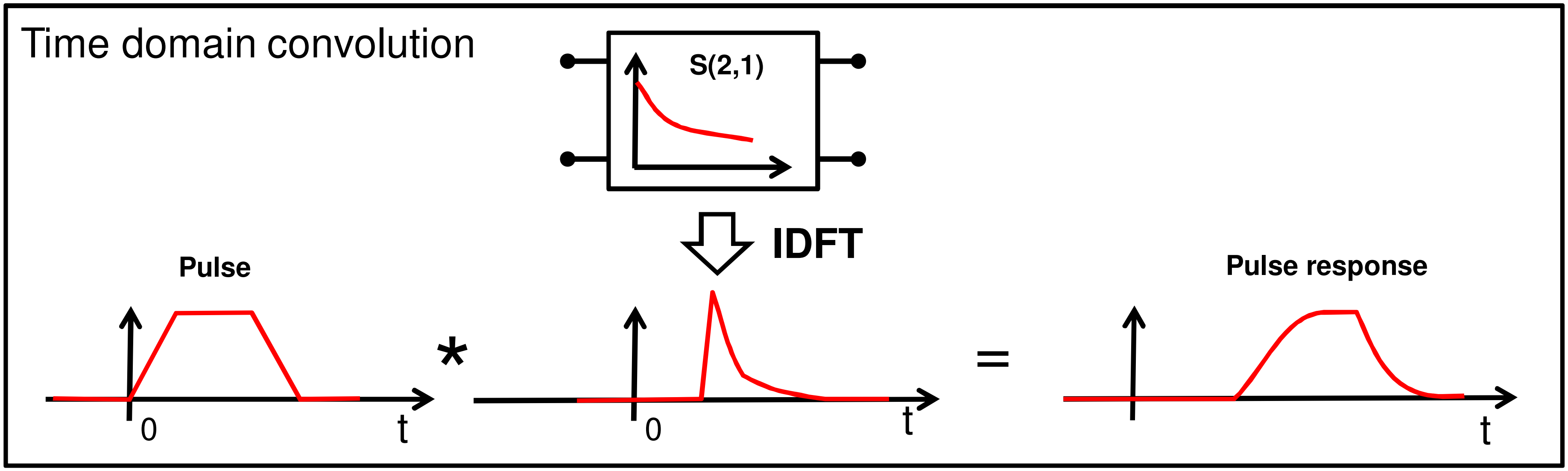
$$v_{s,causal}(t_k) = u_{T_0}(t_k).v_s(t_k)$$

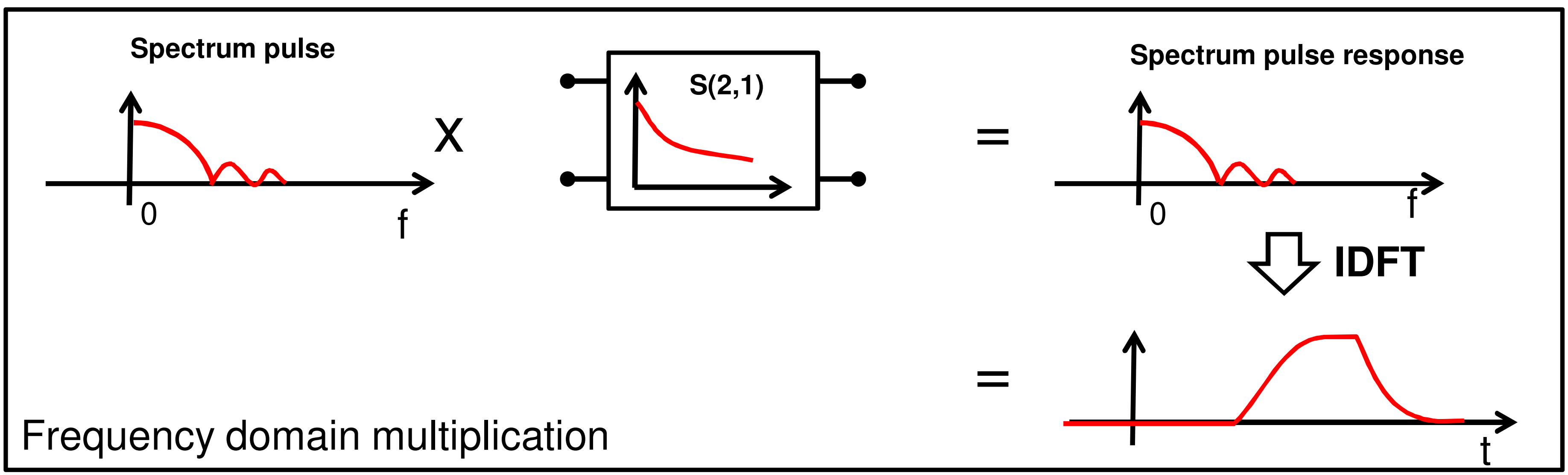


Blind causality enforcement

Calculation of time domain responses

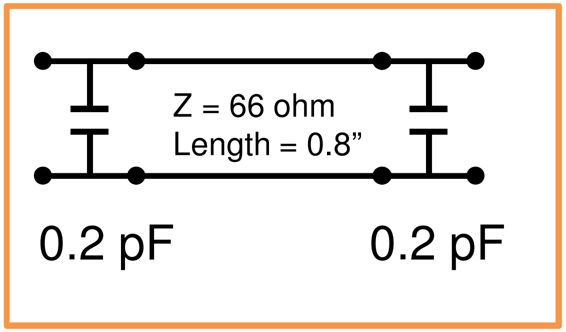
Calculation time domain response



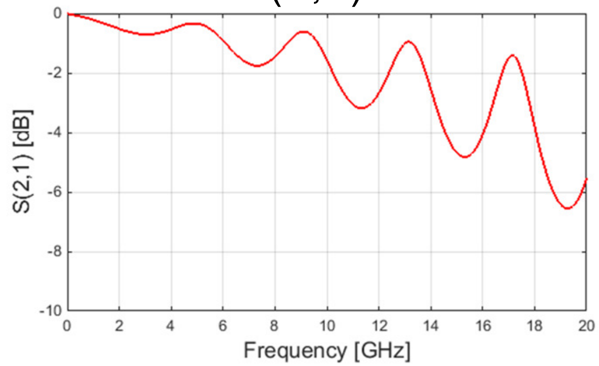


Causality issues due to bandwidth limitation

Time domain convolution

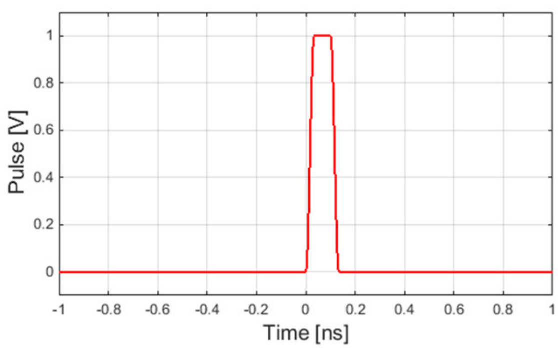


S(2,1)



IDFT

Excitation pulse



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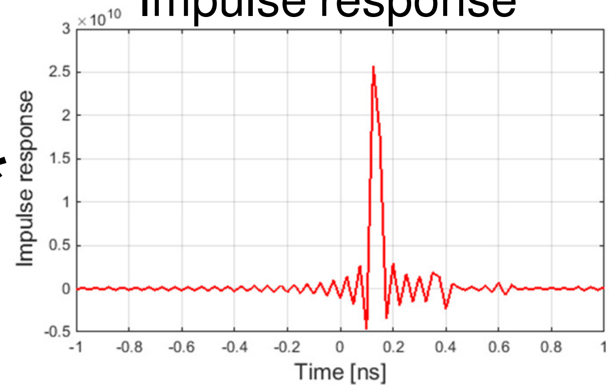
Impulse response



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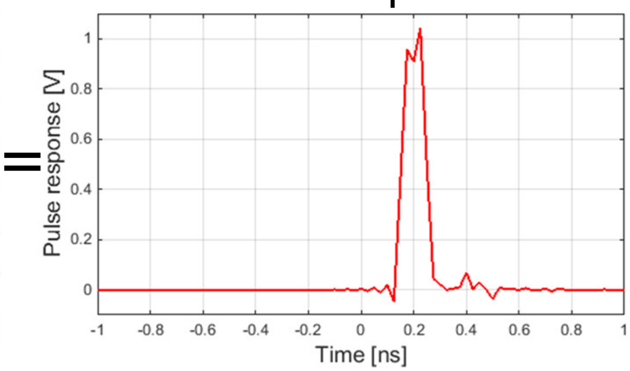
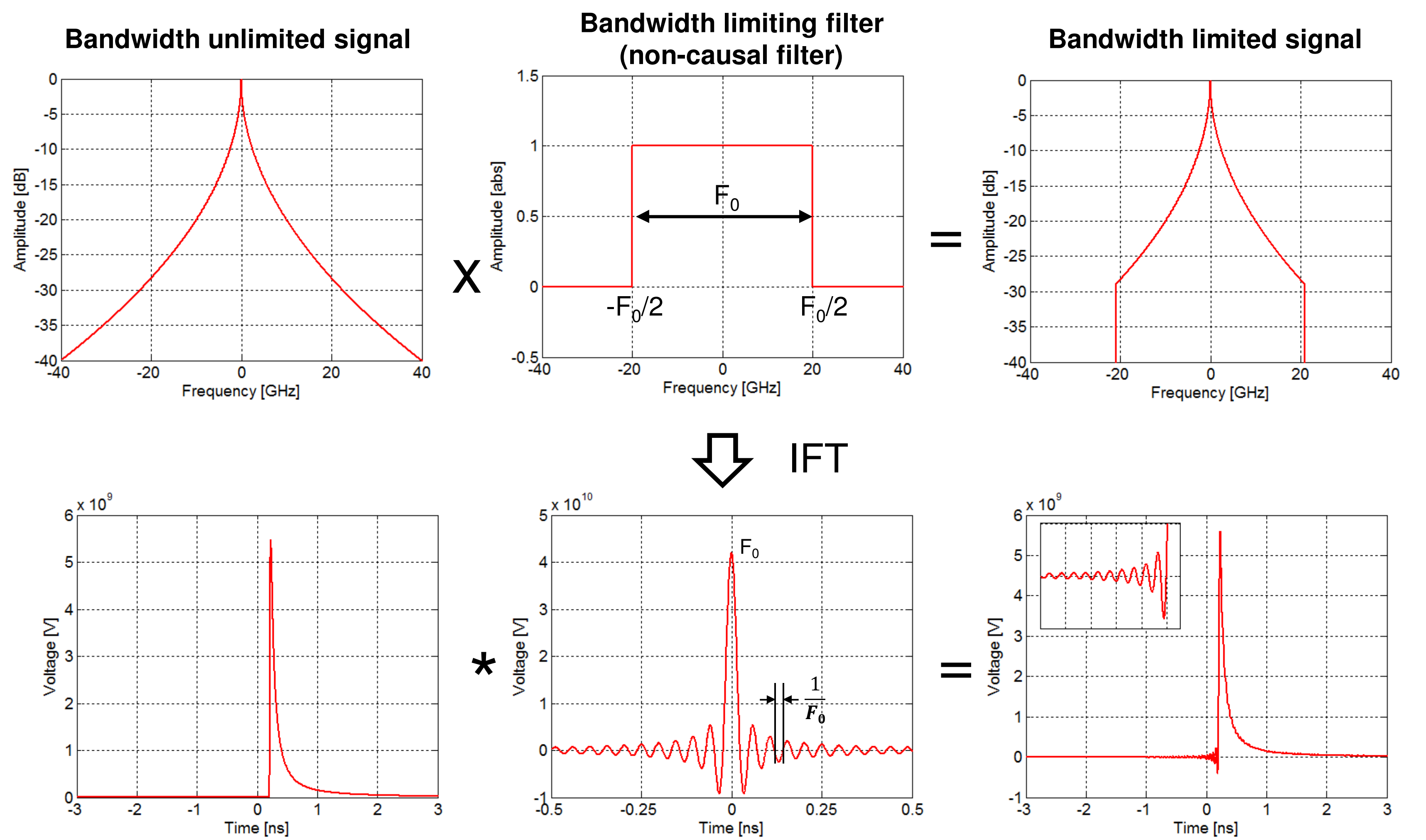
Pulse response



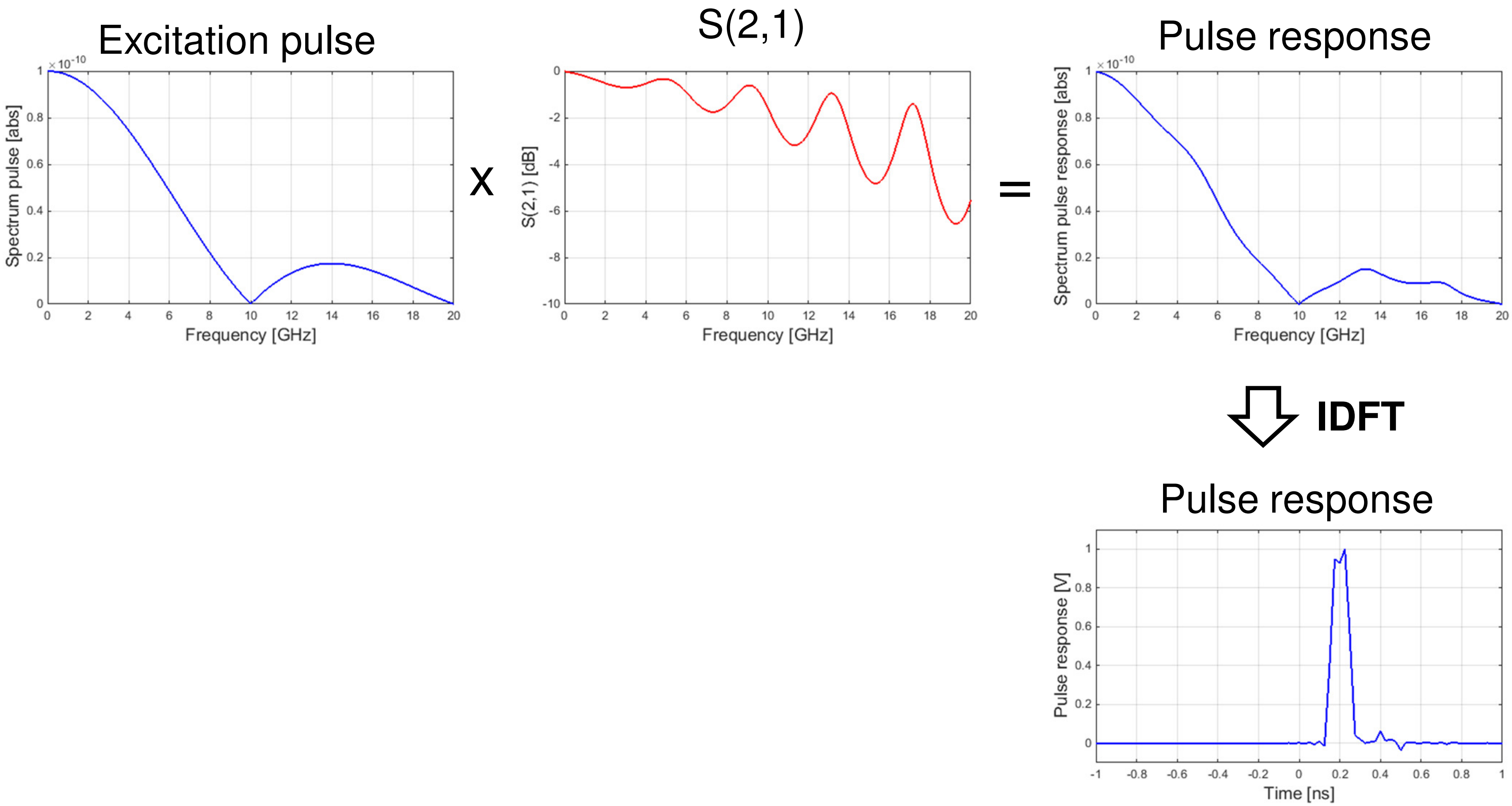
- Impulse response is non-causal: ringing
- Value every $\Delta t = \frac{1}{2.F_{max}}$ = 25 ps

Bandwidth limitation of an S-parameter model

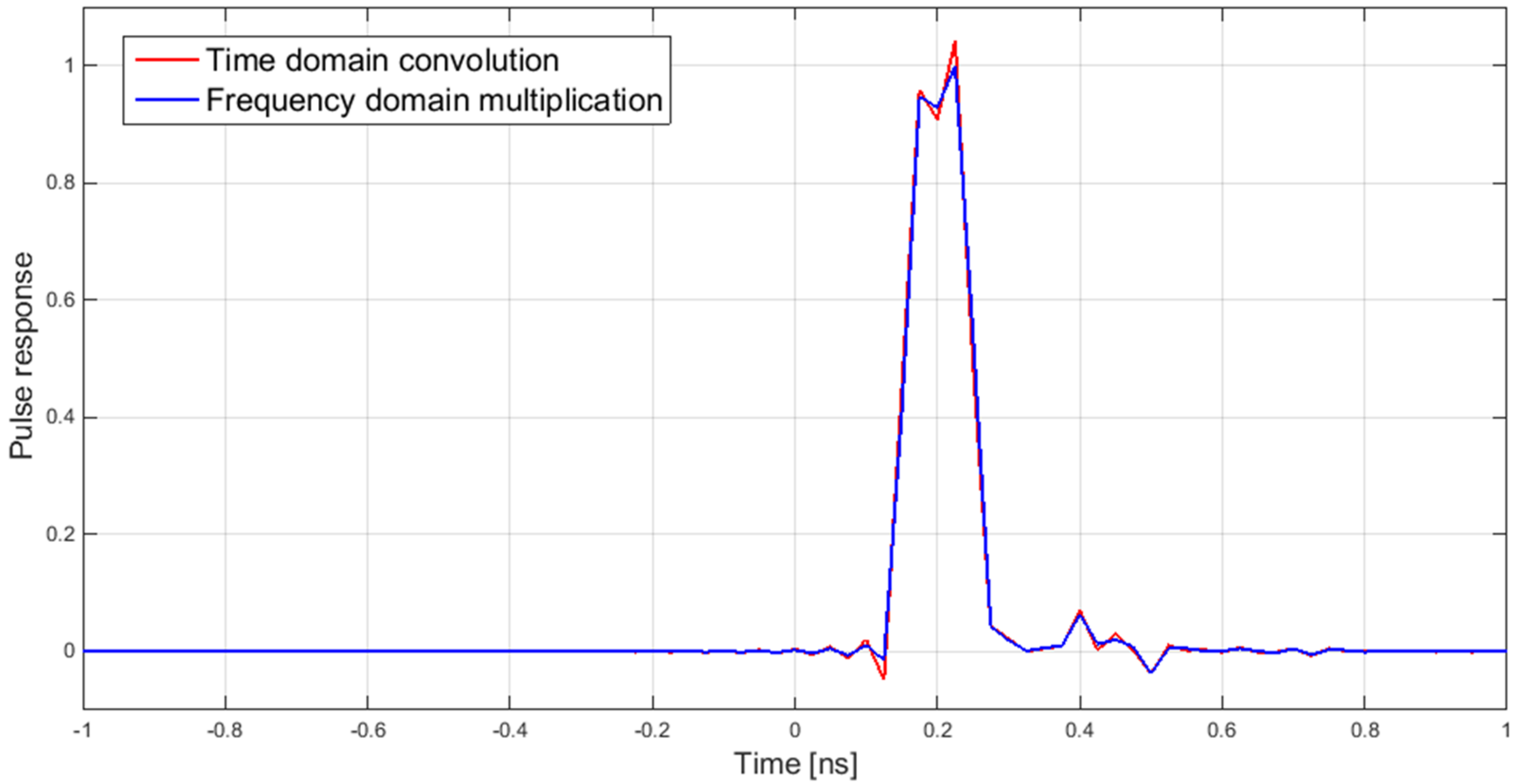


→ Gibbs phenomenon

Frequency domain multiplication

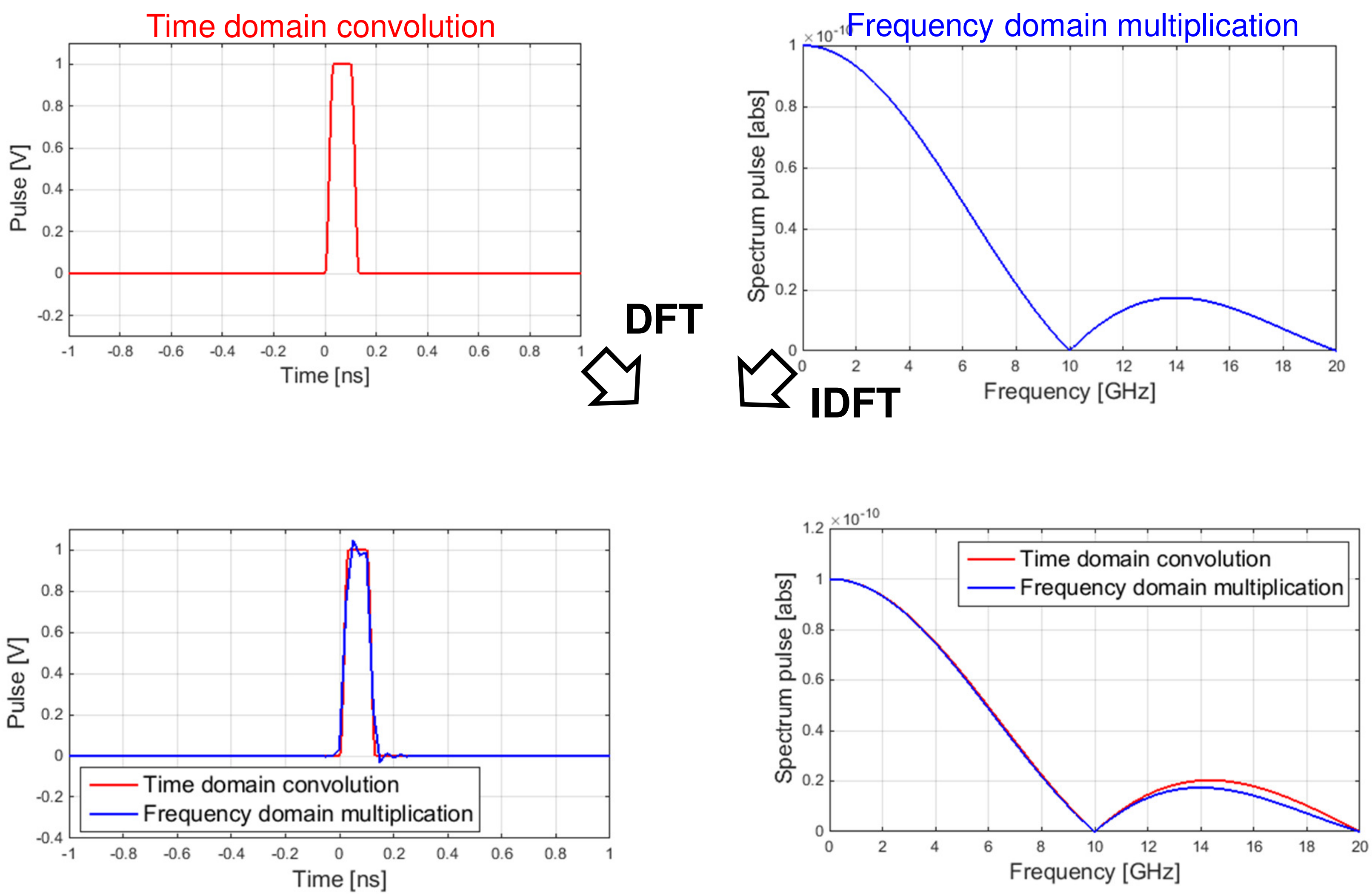


Frequency domain multiplication vs. time domain convolution

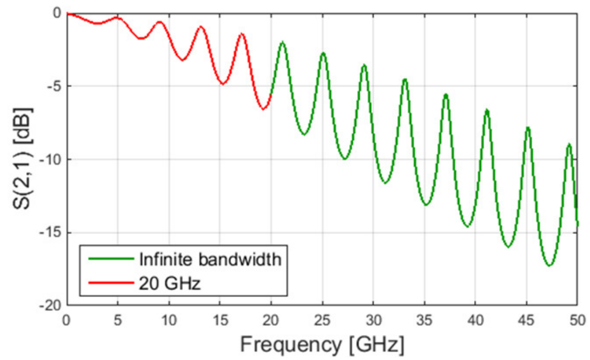


Methods do not match perfectly!

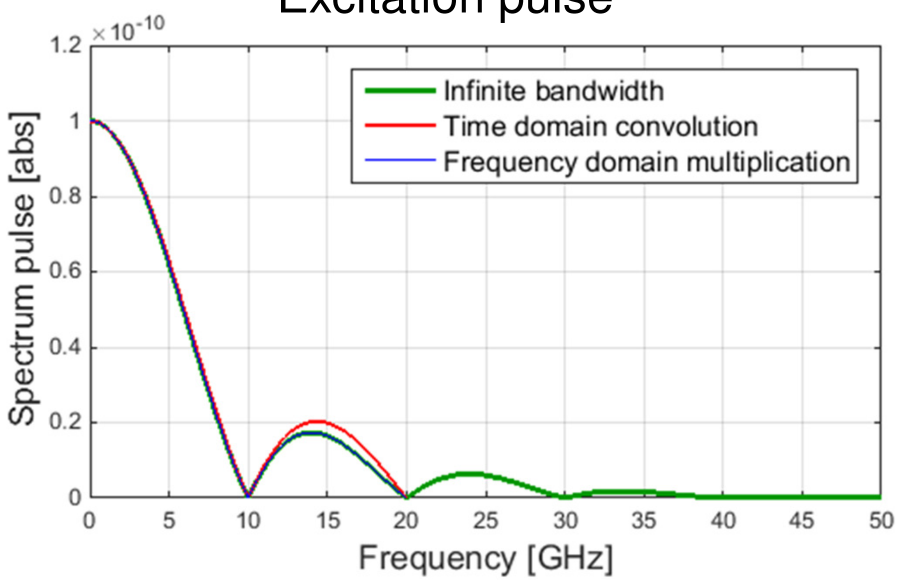
Frequency domain multiplication vs. time domain convolution



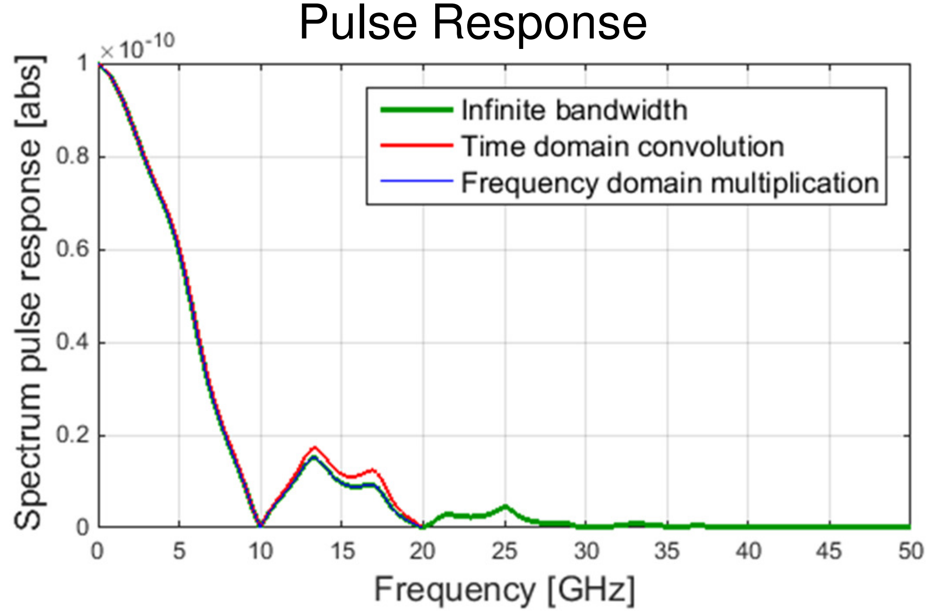
Frequency domain multiplication vs. time domain convolution



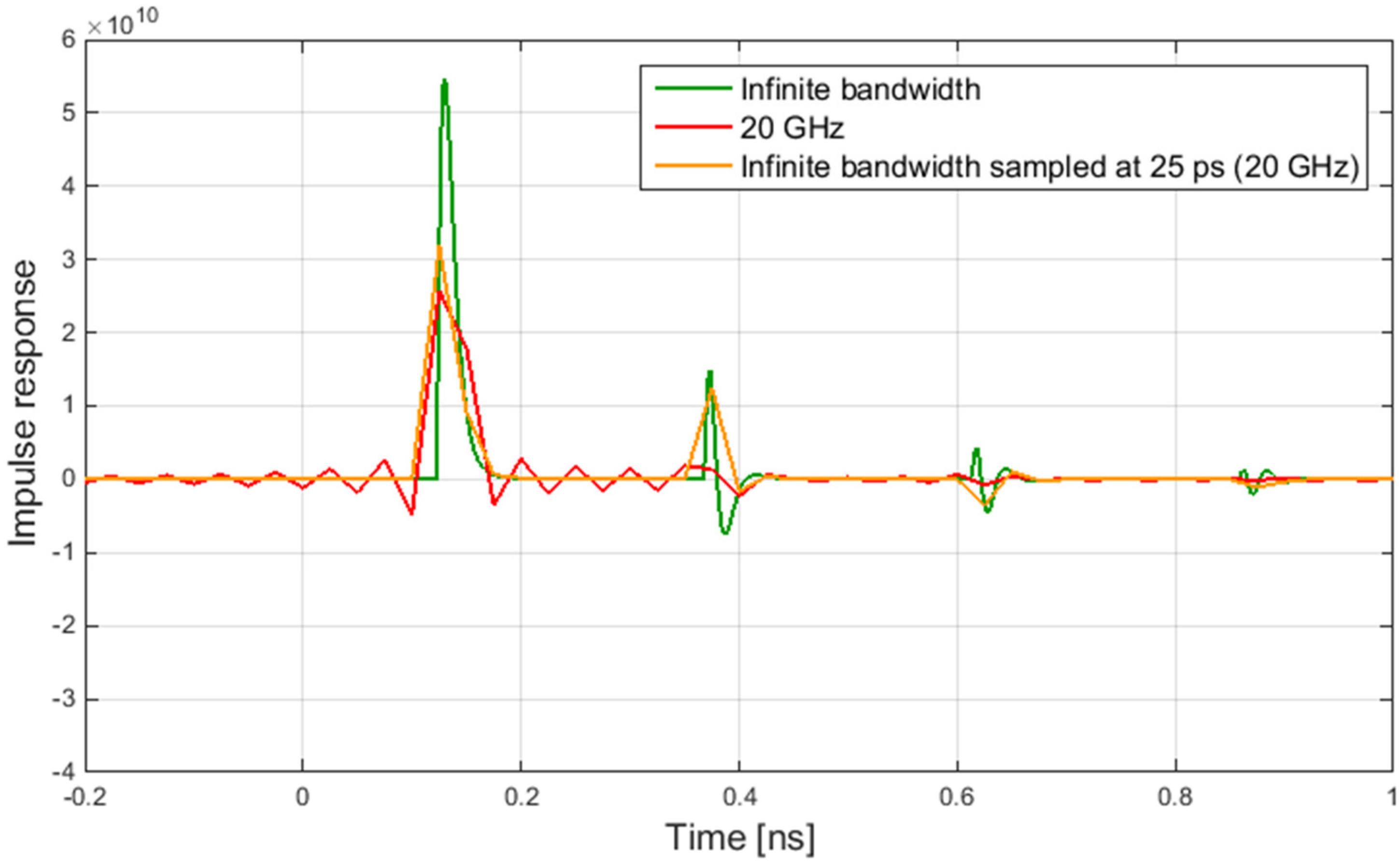
Excitation pulse



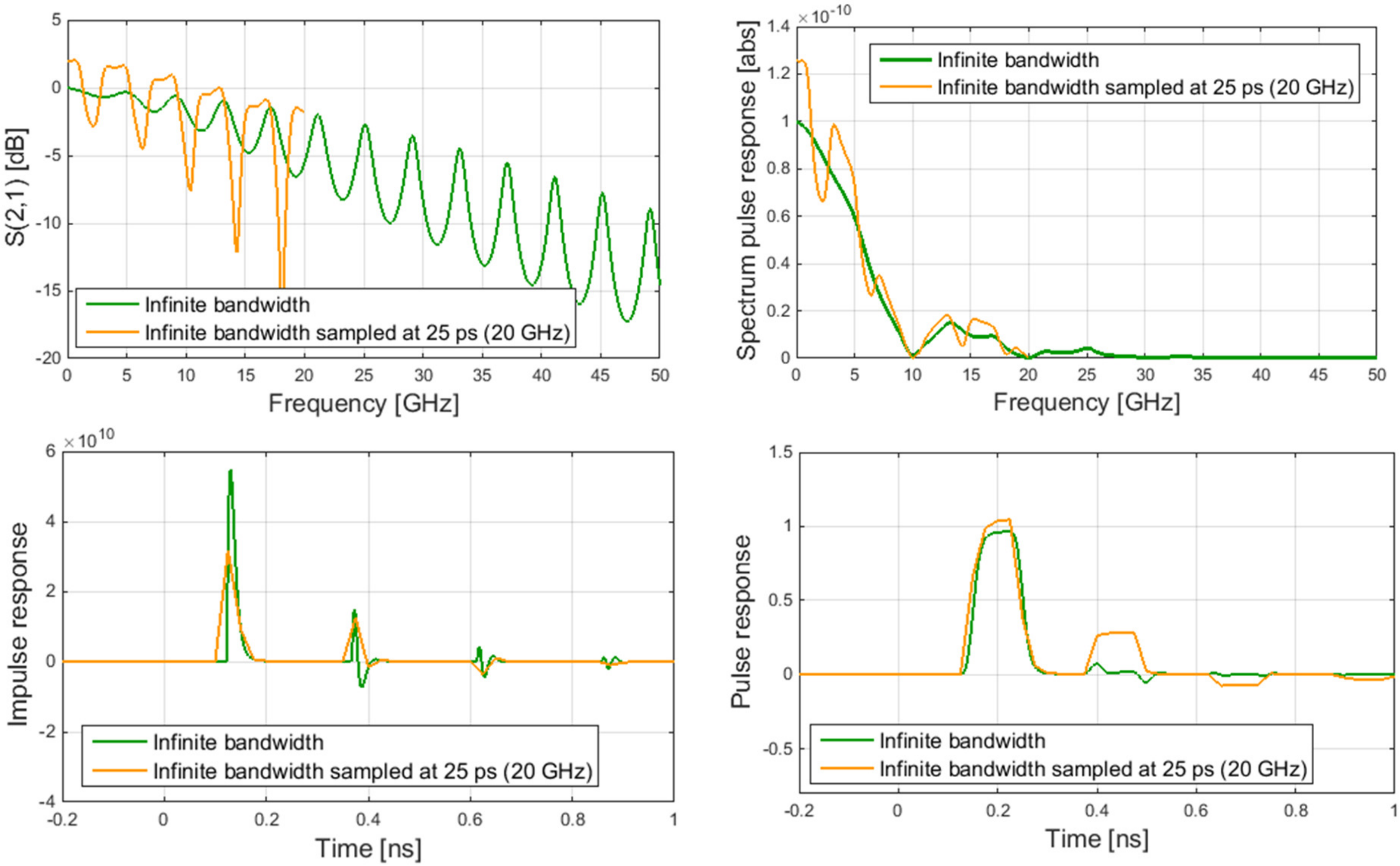
Pulse Response



Frequency domain multiplication vs. time domain convolution

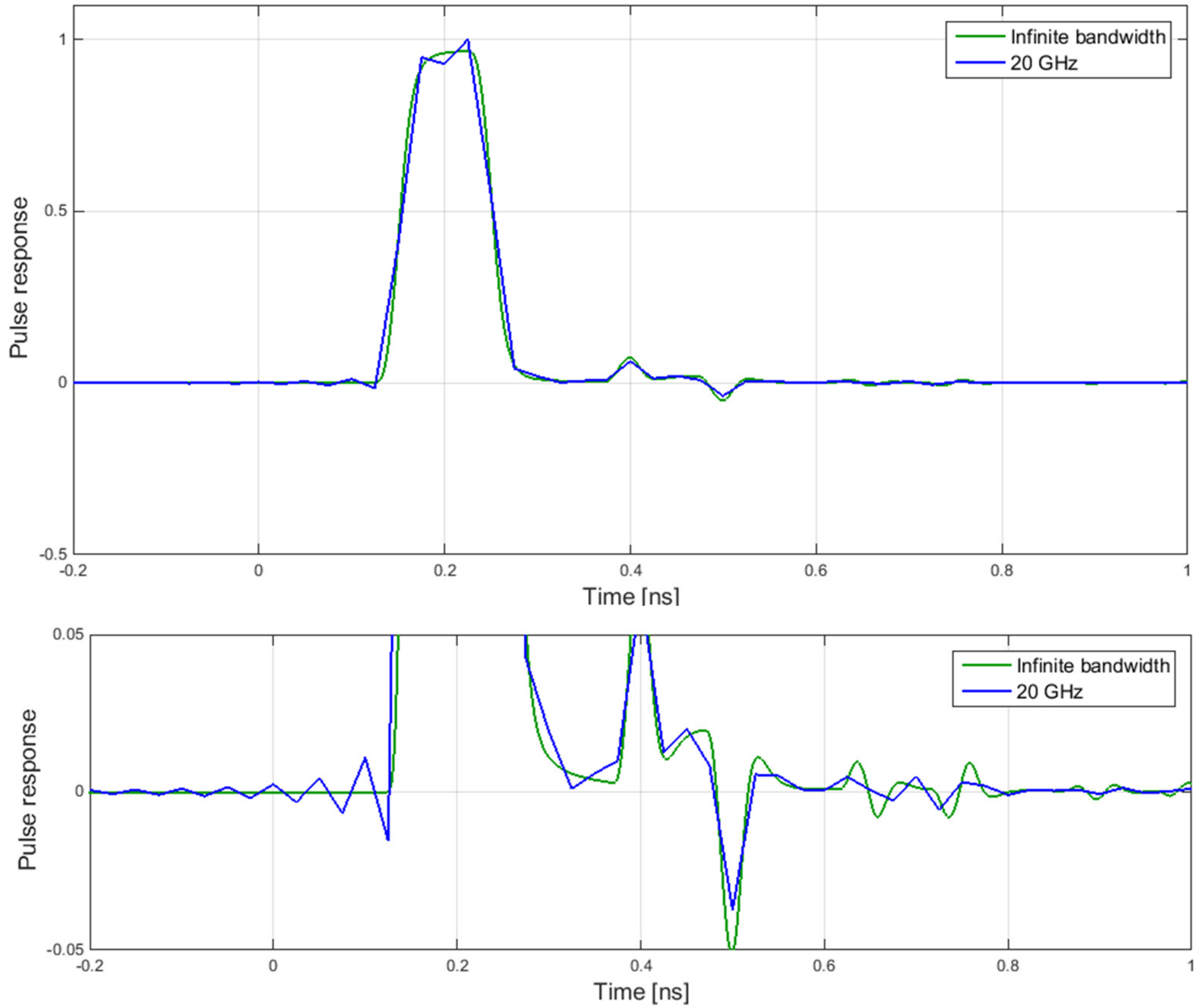


Frequency domain multiplication vs. time domain convolution



Time domain sampling, when not done properly (Nyquist theorem) will introduce frequency domain leakage or aliasing

Bandwidth limitation causes ringing and loss of resolution



Bandwidth limitation causes ringing and loss of resolution

To reduce the ringing: need more bandwidth

To have more time points per bit: need more bandwidth

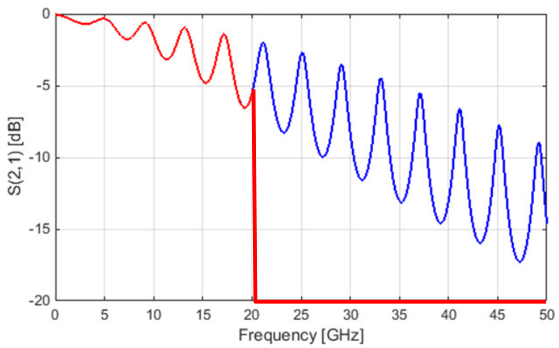
How much bandwidth is needed ?

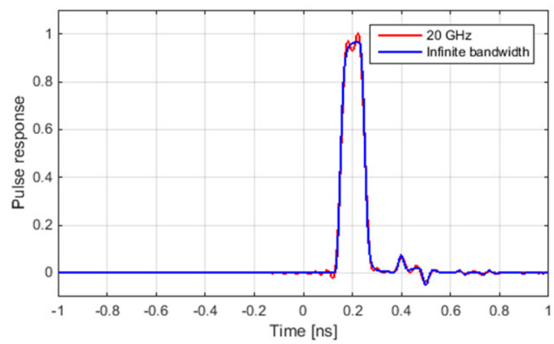
Minimum N points per bit required: $$F_{max} \geq \frac{N.Bitrate}{2}$$

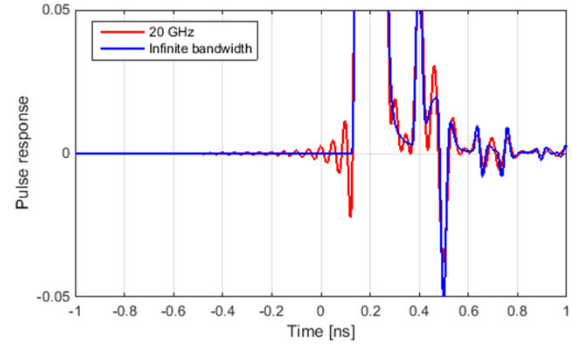
Bitrate [Gb/s]	Bit time [ps]	N	Δt [ps]	Fmax [GHz]
10	100	4	25.00	20
10	100	10	10.00	50
10	100	20	5.00	100
10	100	50	2.00	250
10	100	100	1.00	500
25	40	4	10.00	50
25	40	10	4.00	125
25	40	20	2.00	250
25	40	50	0.80	625
25	40	100	0.40	1250

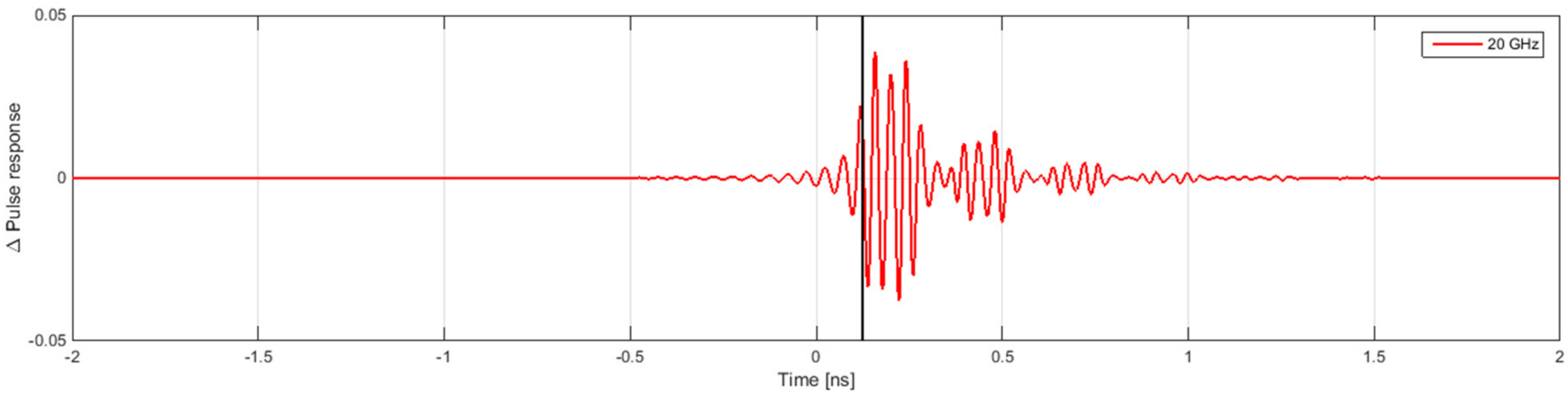
Up to which frequency is actual data required?
When can I apply zero-padding?

Max bandwidth required



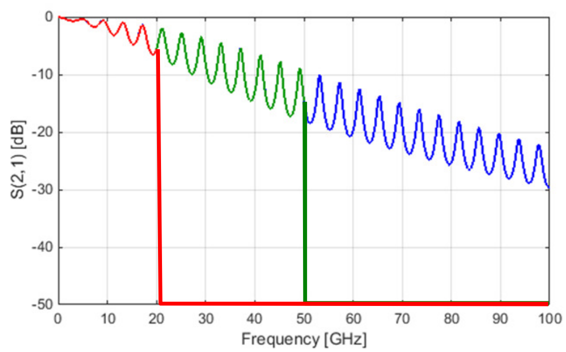


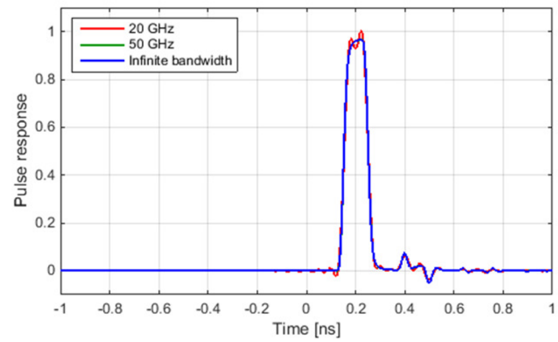


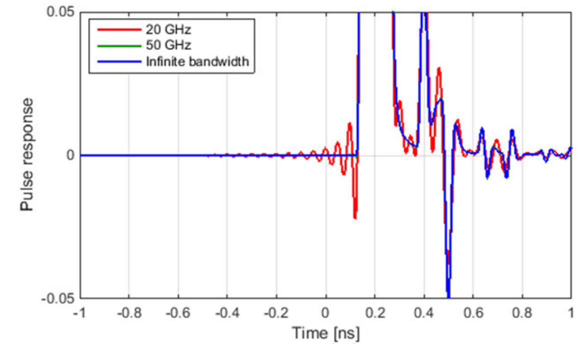


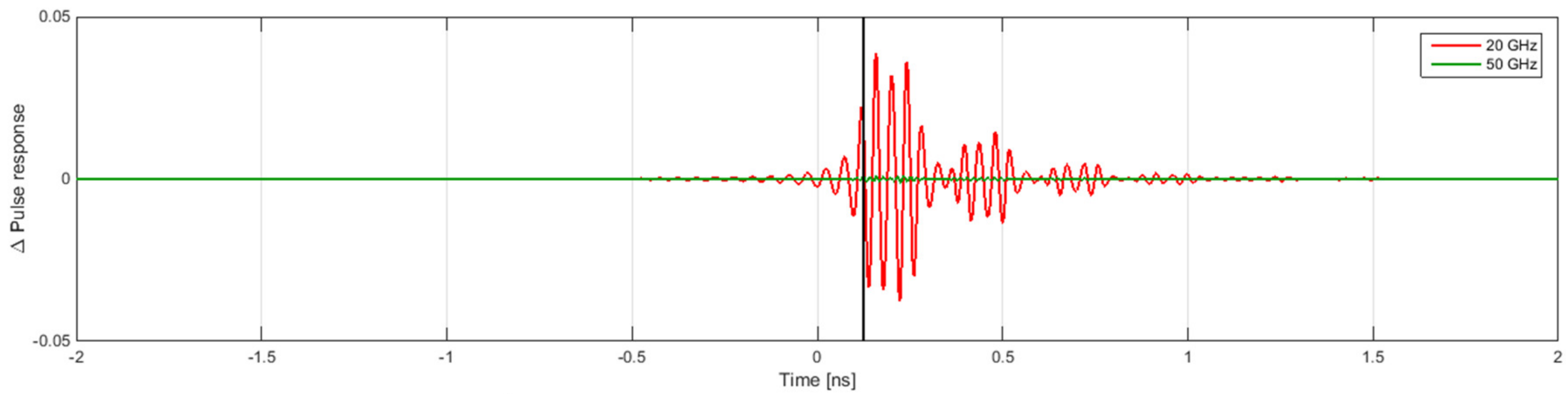
Notice: Main part of ringing in causal part of pulse response!

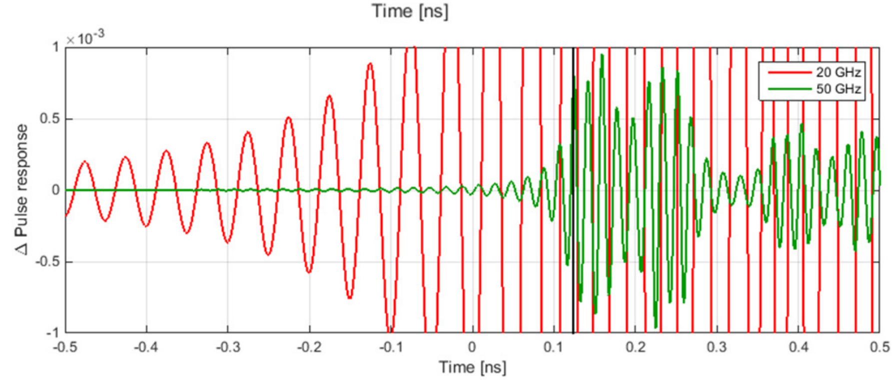
Max bandwidth required



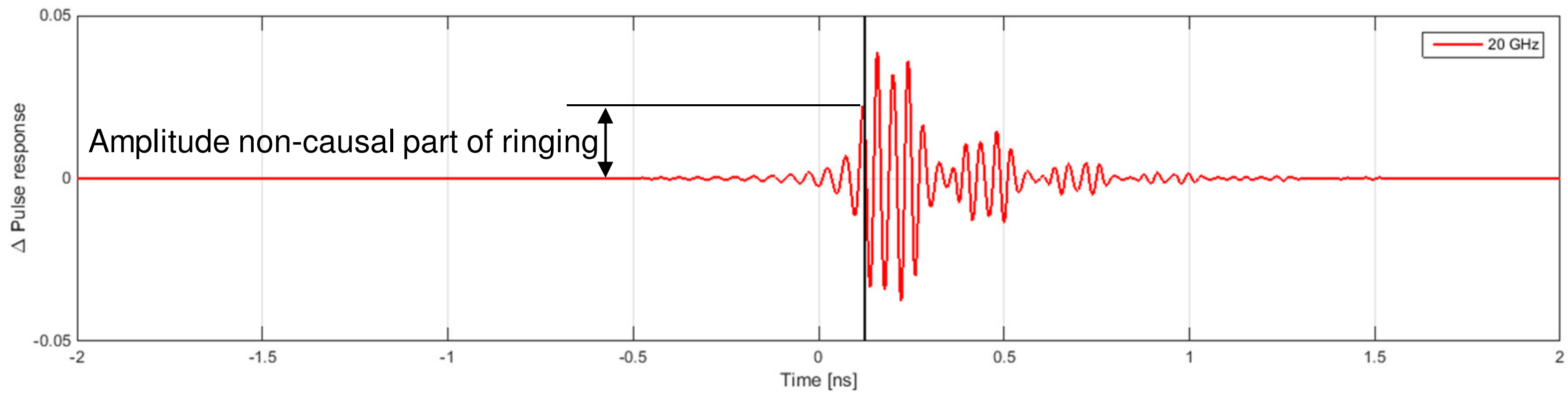


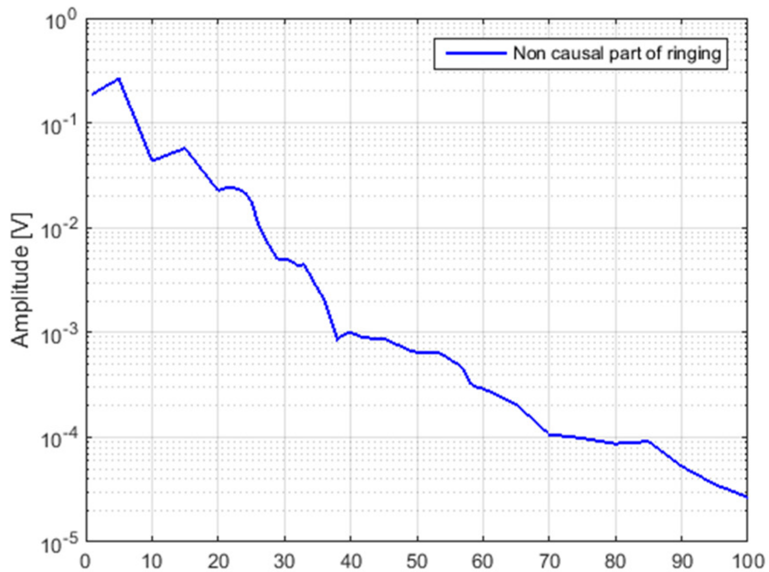






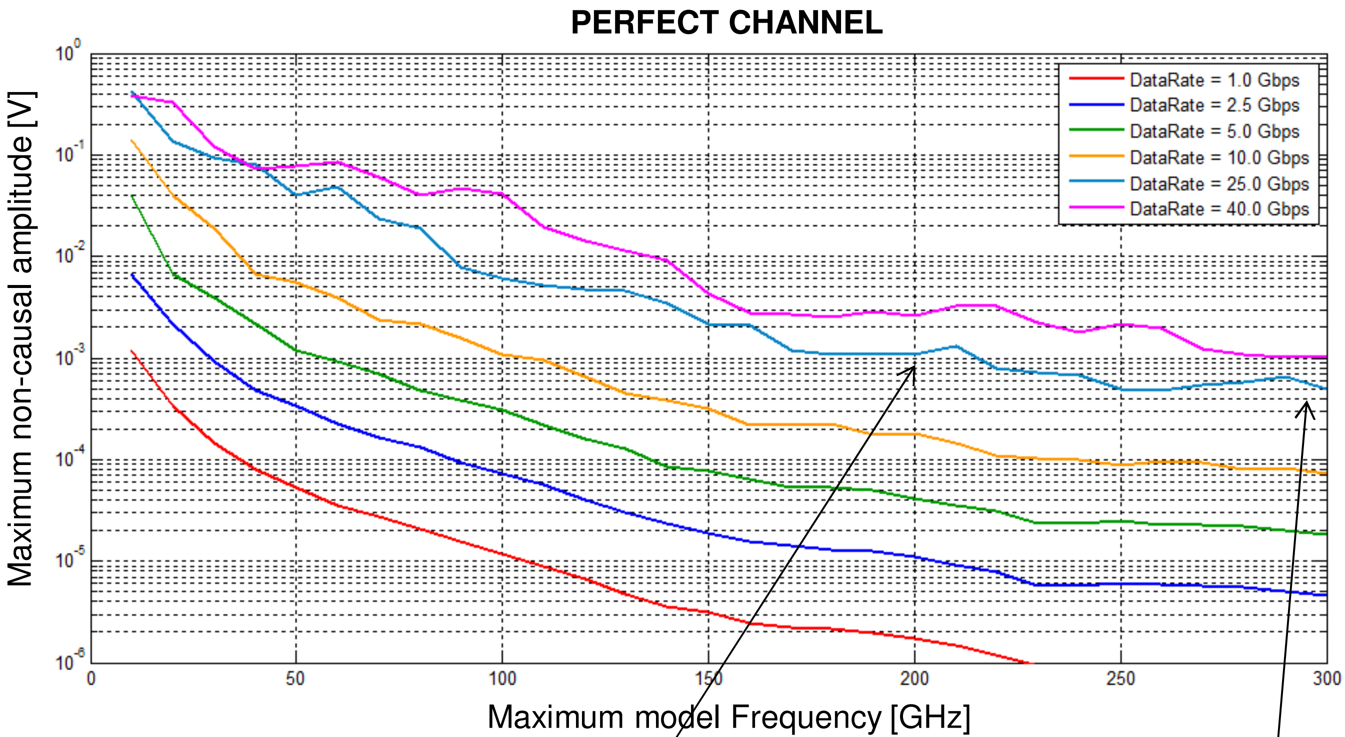
Max bandwidth required





Maximum model frequency [GHz]

Max bandwidth required

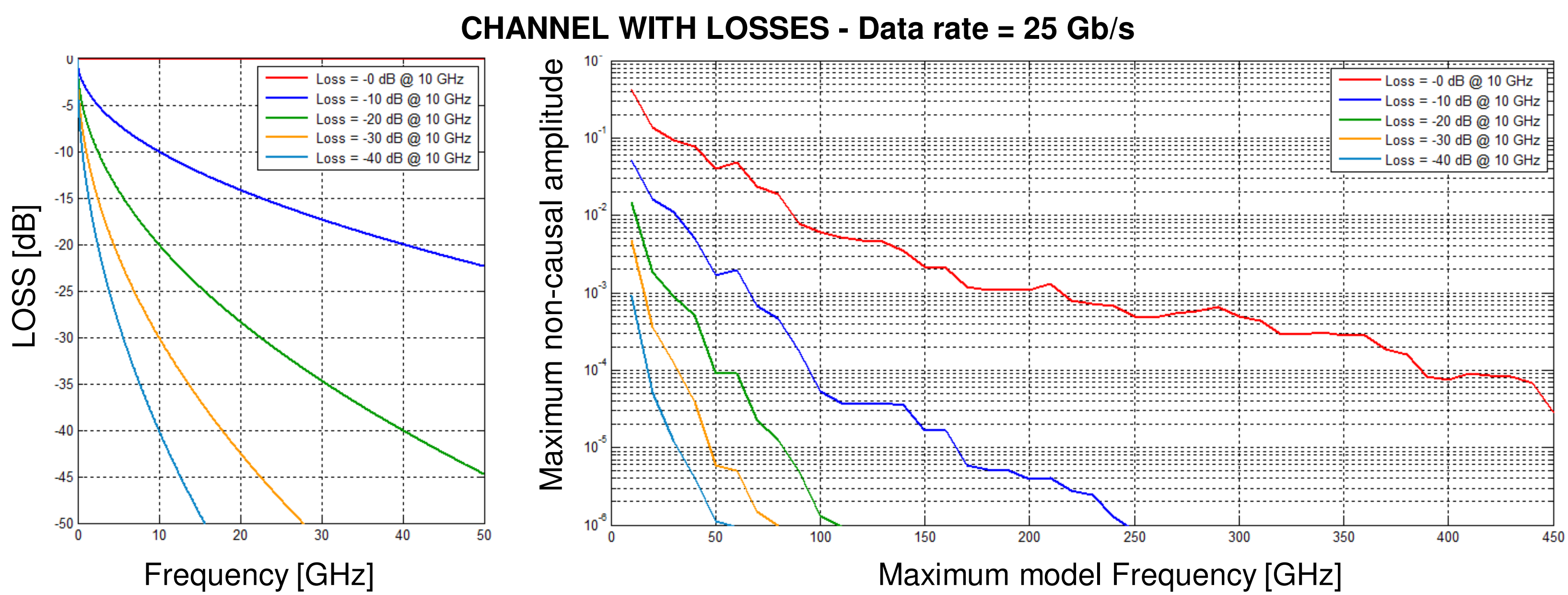


Maximum non-causal amplitude < 1e-3:
Maximum model frequency = 200 GHz for data rate = 25 Gb/s

Maximum non-causal amplitude < 1e-4:
Maximum model frequency > 300 GHz for data rate = 25 ,40 Gb/s

Perfect channel defines upper limit for bandwidth.

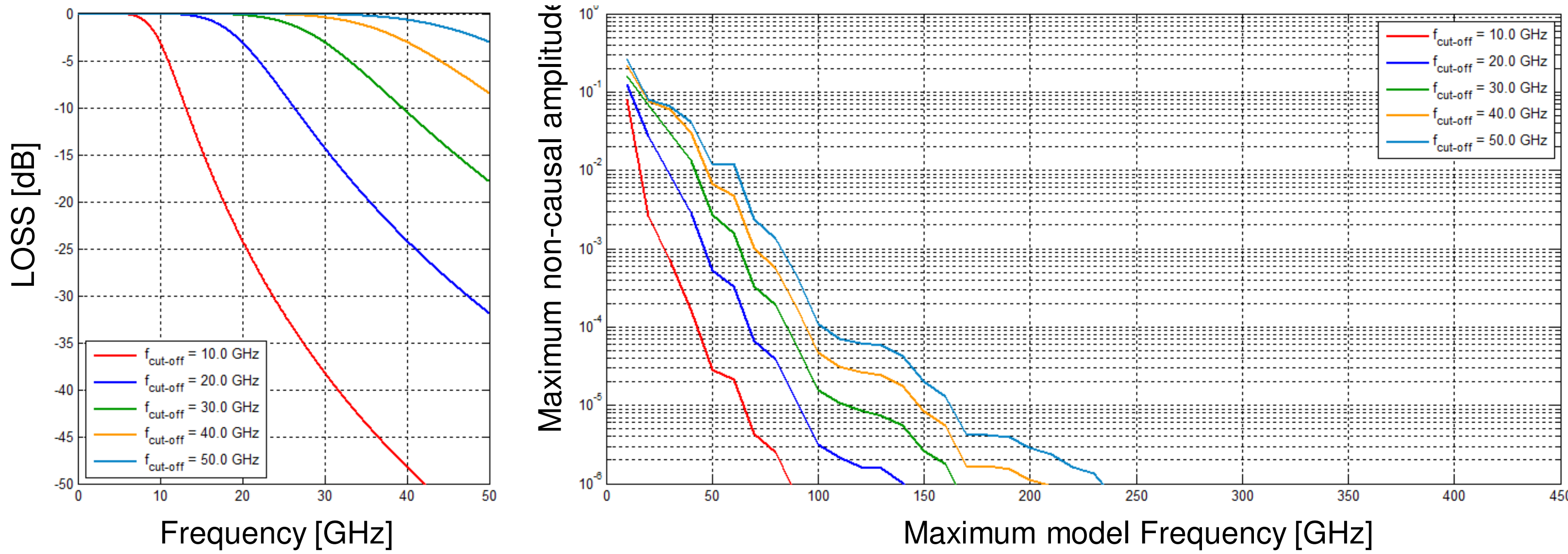
Max bandwidth required



Maximum model frequency F_{max} drops significantly if channel is lossy.

Max bandwidth required

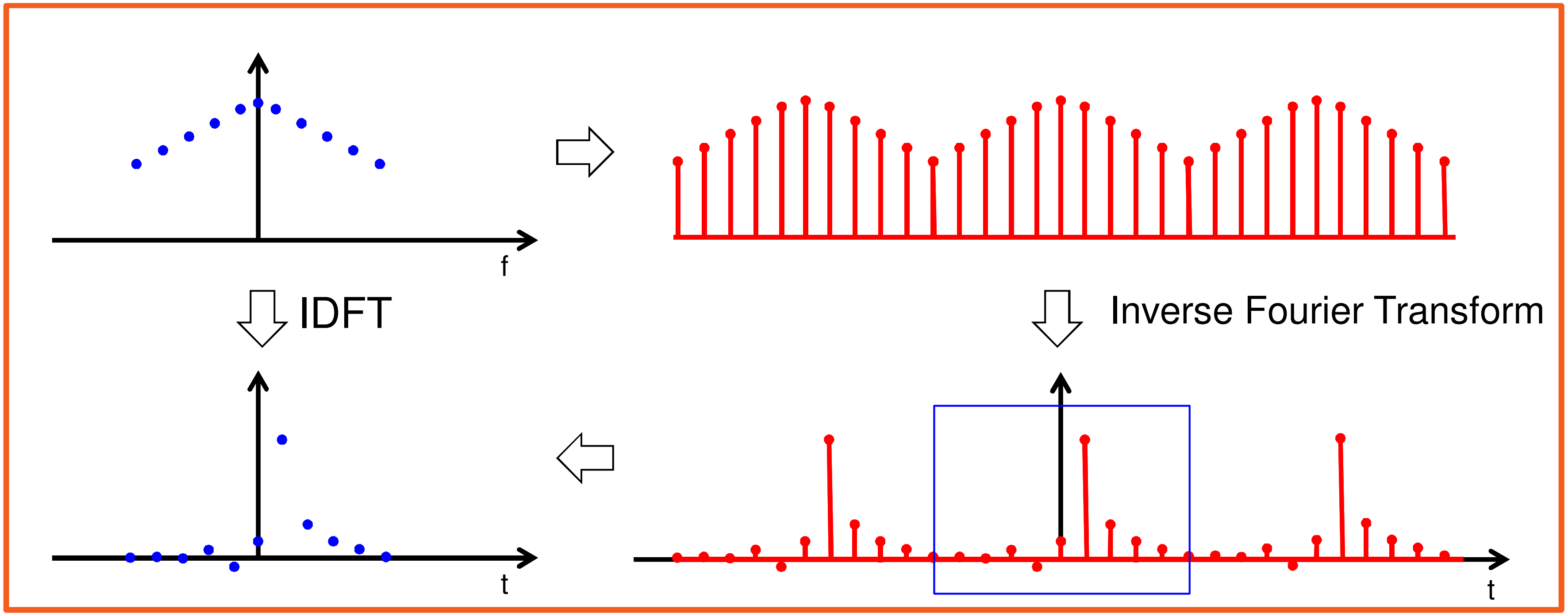
PERFECT CHANNEL, BANDWIDTH LIMITED BY BUTTERWORTH FILTER - Data rate = 25 Gb/s



Maximum model frequency F_{max} drops further.

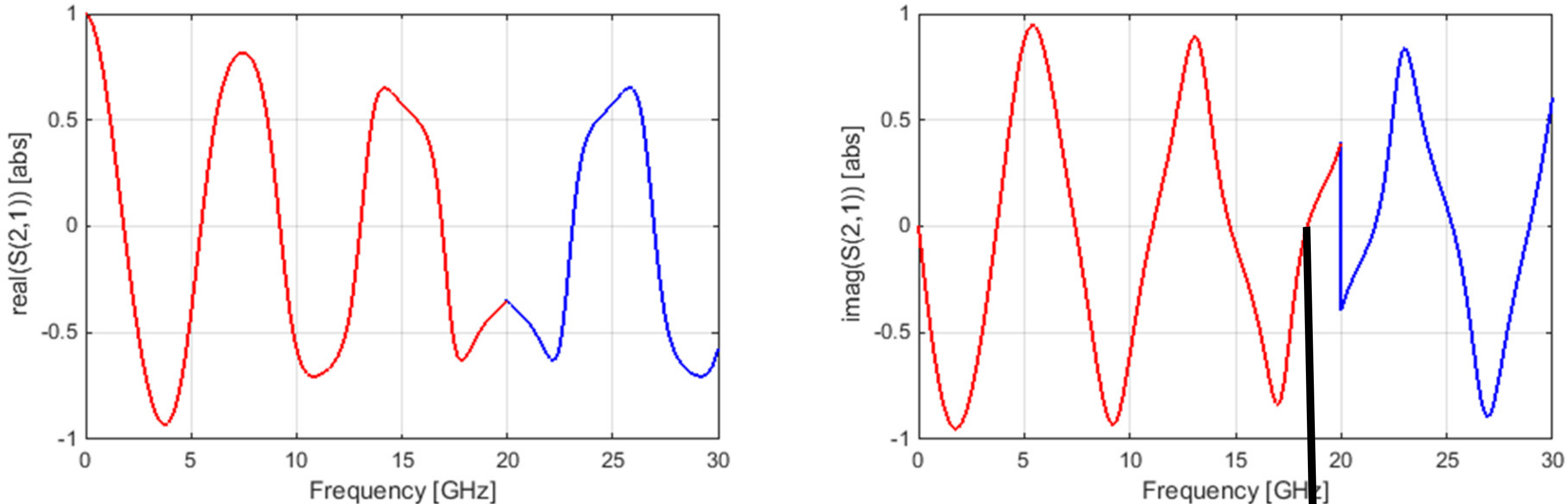
Techniques to minimize ringing

Discrete Fourier Transform vs Fourier Transform

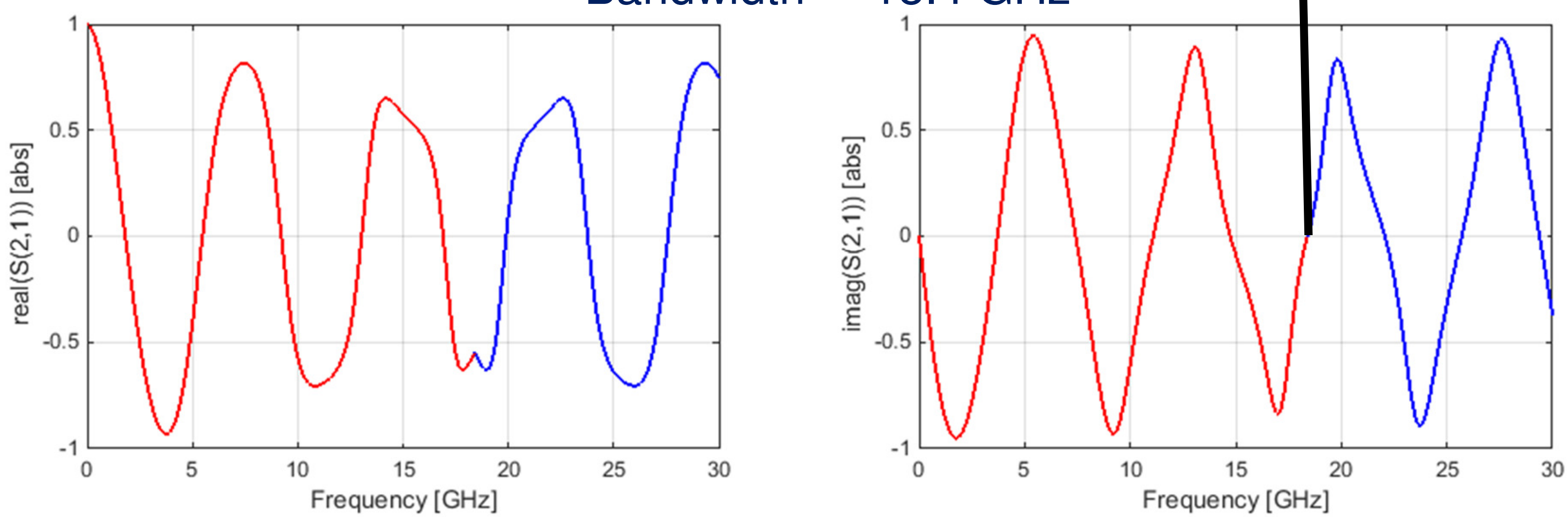


Techniques to minimize ringing - method 1

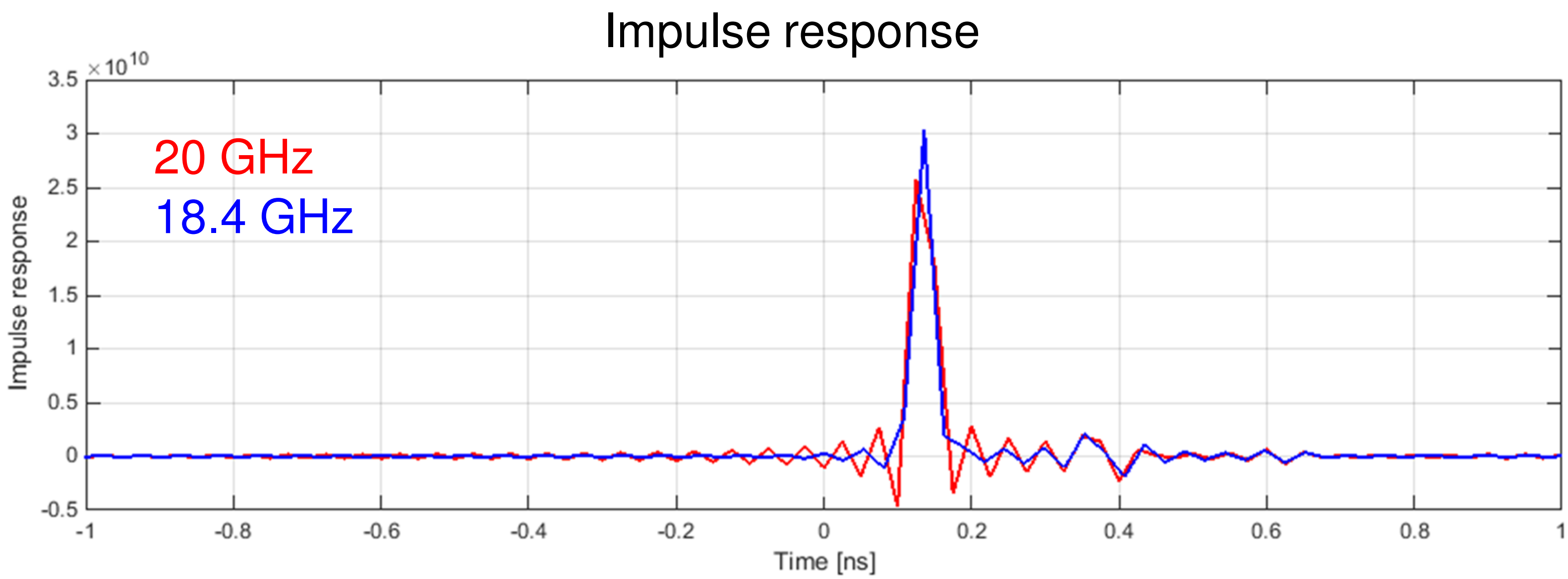
Bandwidth = 20 GHz

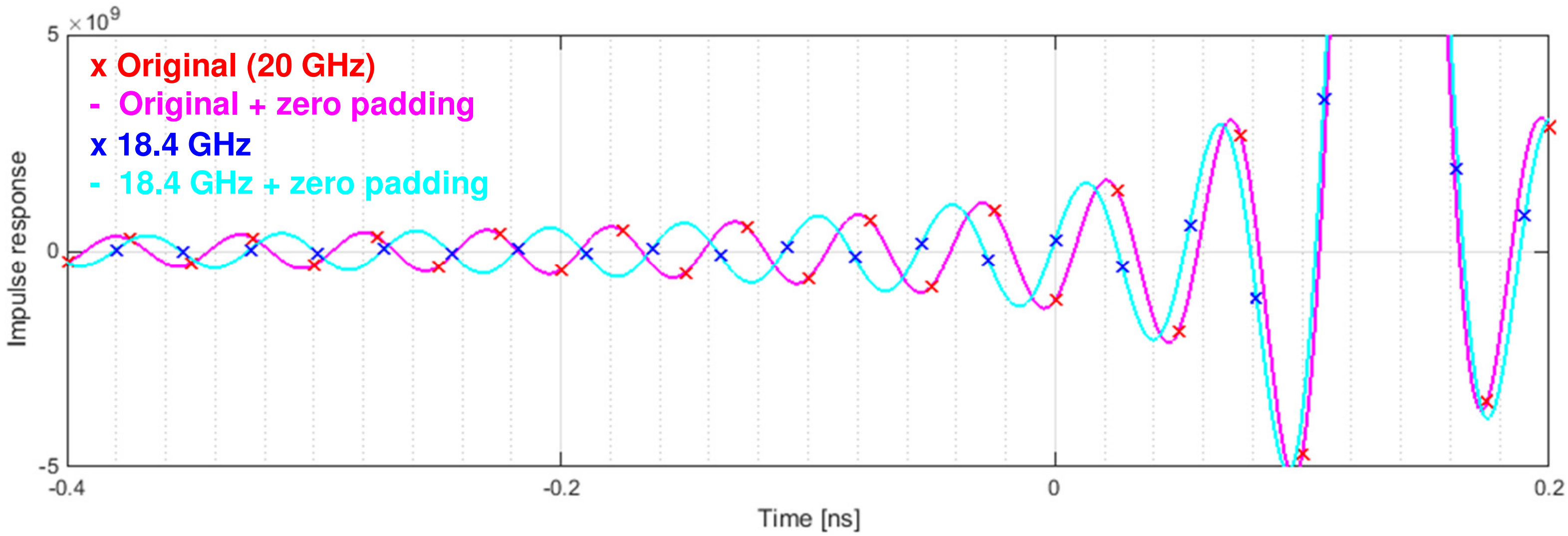


Bandwidth = 18.4 GHz



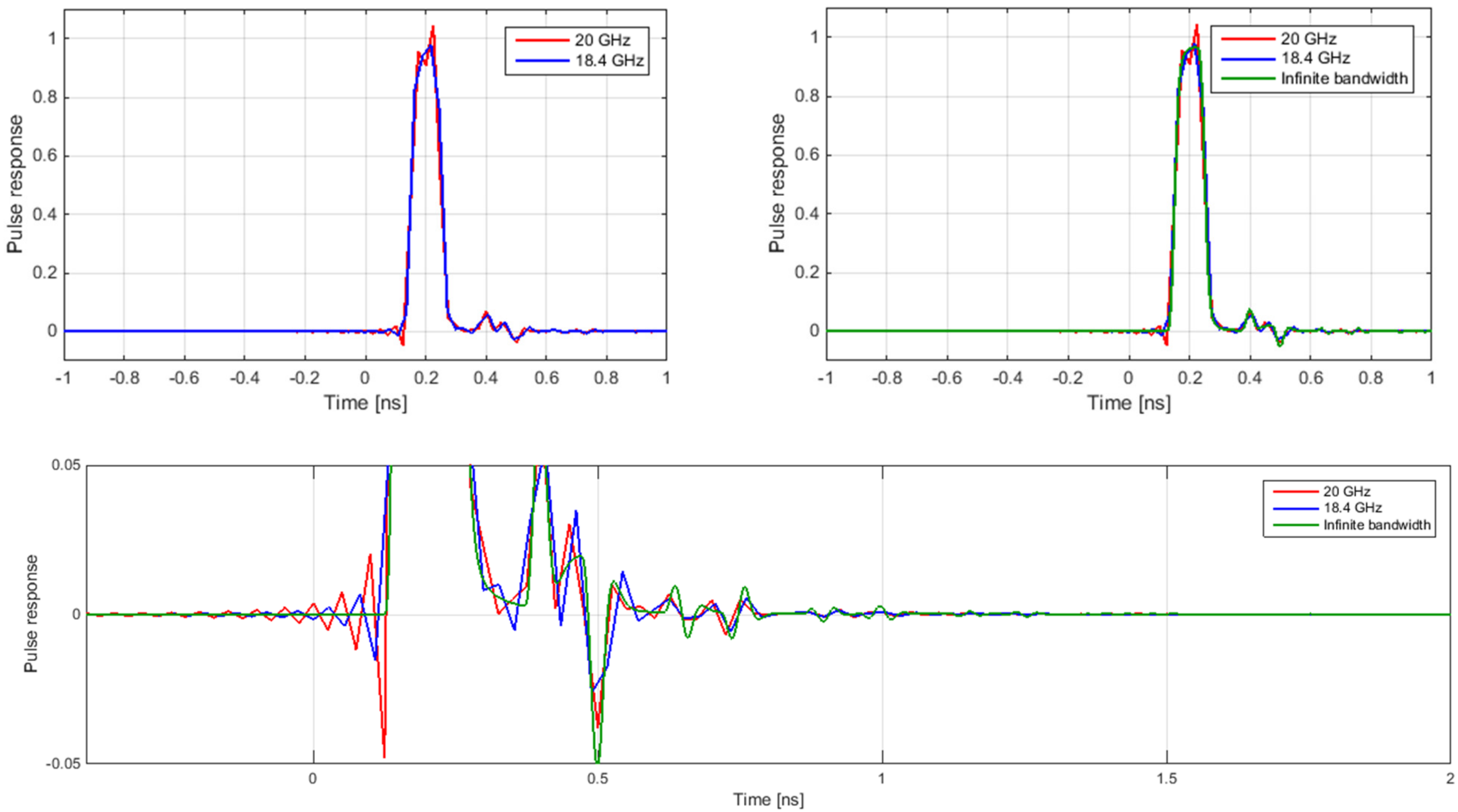
Techniques to minimize ringing - method 1





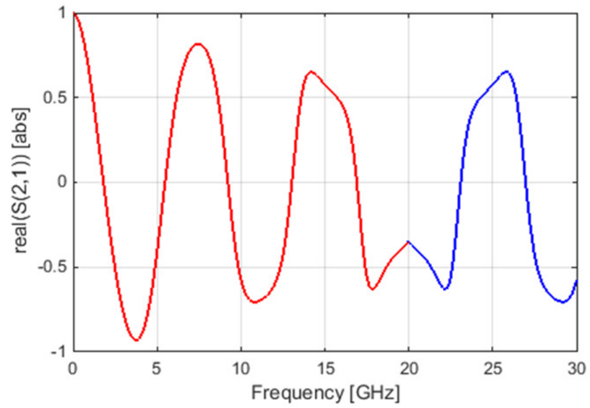
Techniques to minimize ringing - method 1

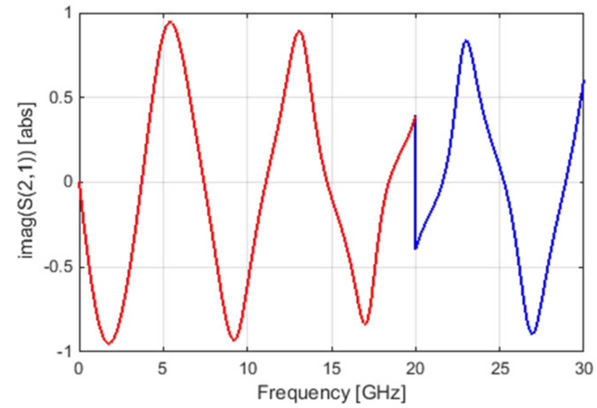
Pulse response



Techniques to minimize ringing - method 2

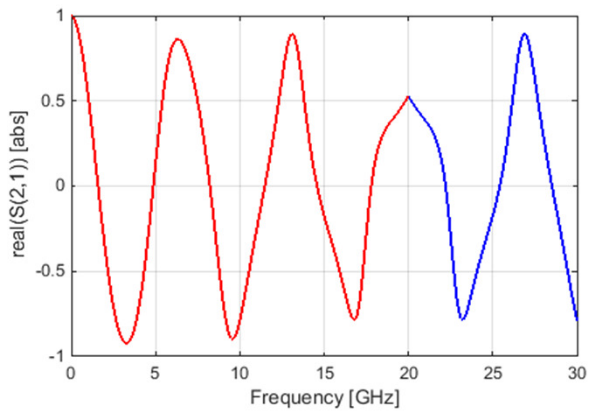
Bandwidth = 20 GHz

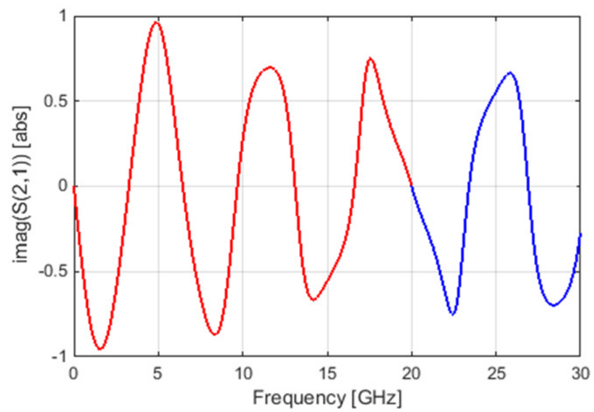




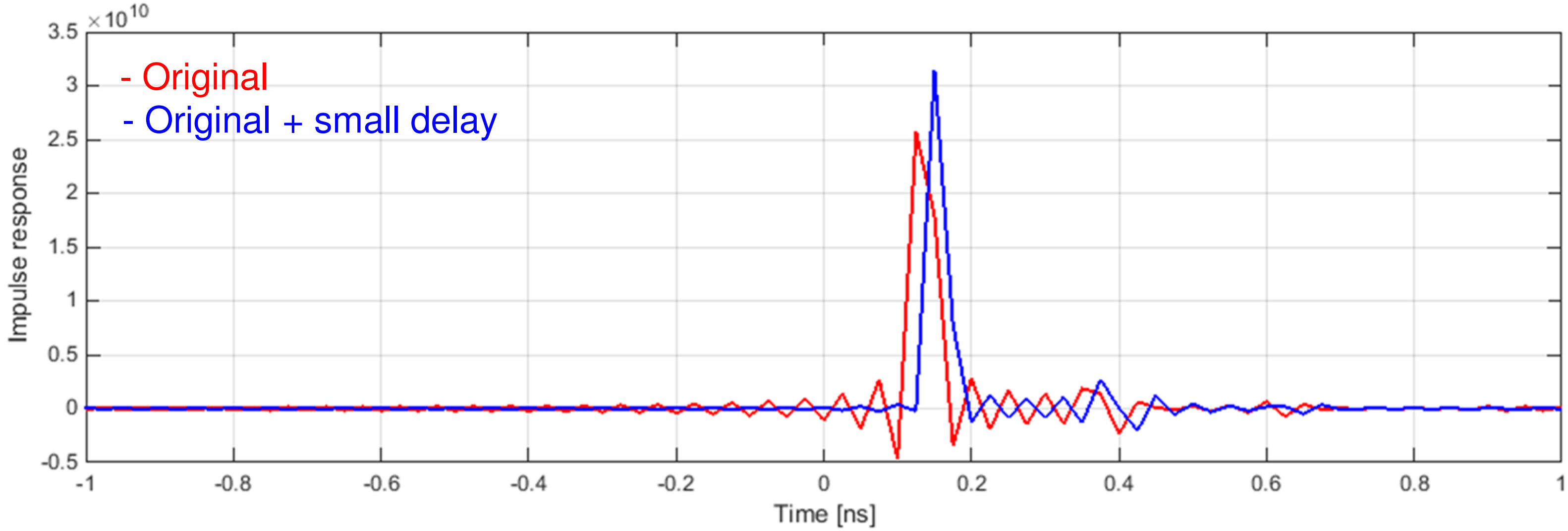
$$S_{2,1}^{new}(f) = S_{2,1}(f).e^{-j\omega\tau} \quad \tau = \alpha\Delta t \quad 0 \leq \alpha < 1$$

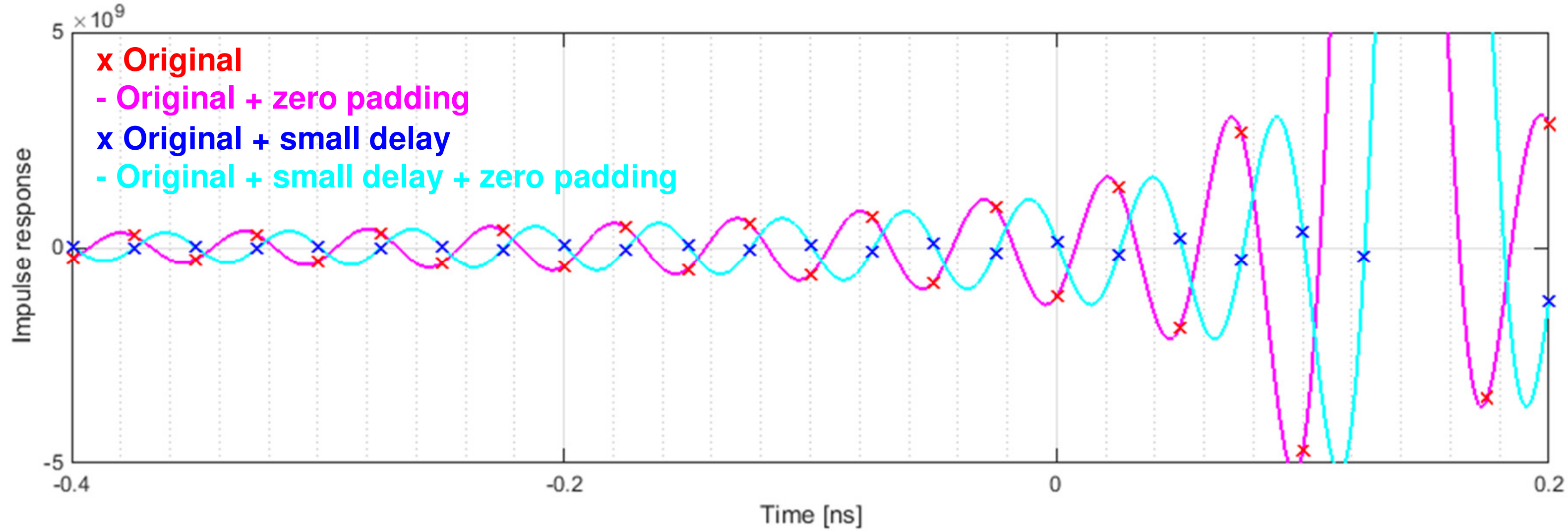
Bandwidth = 20 GHz



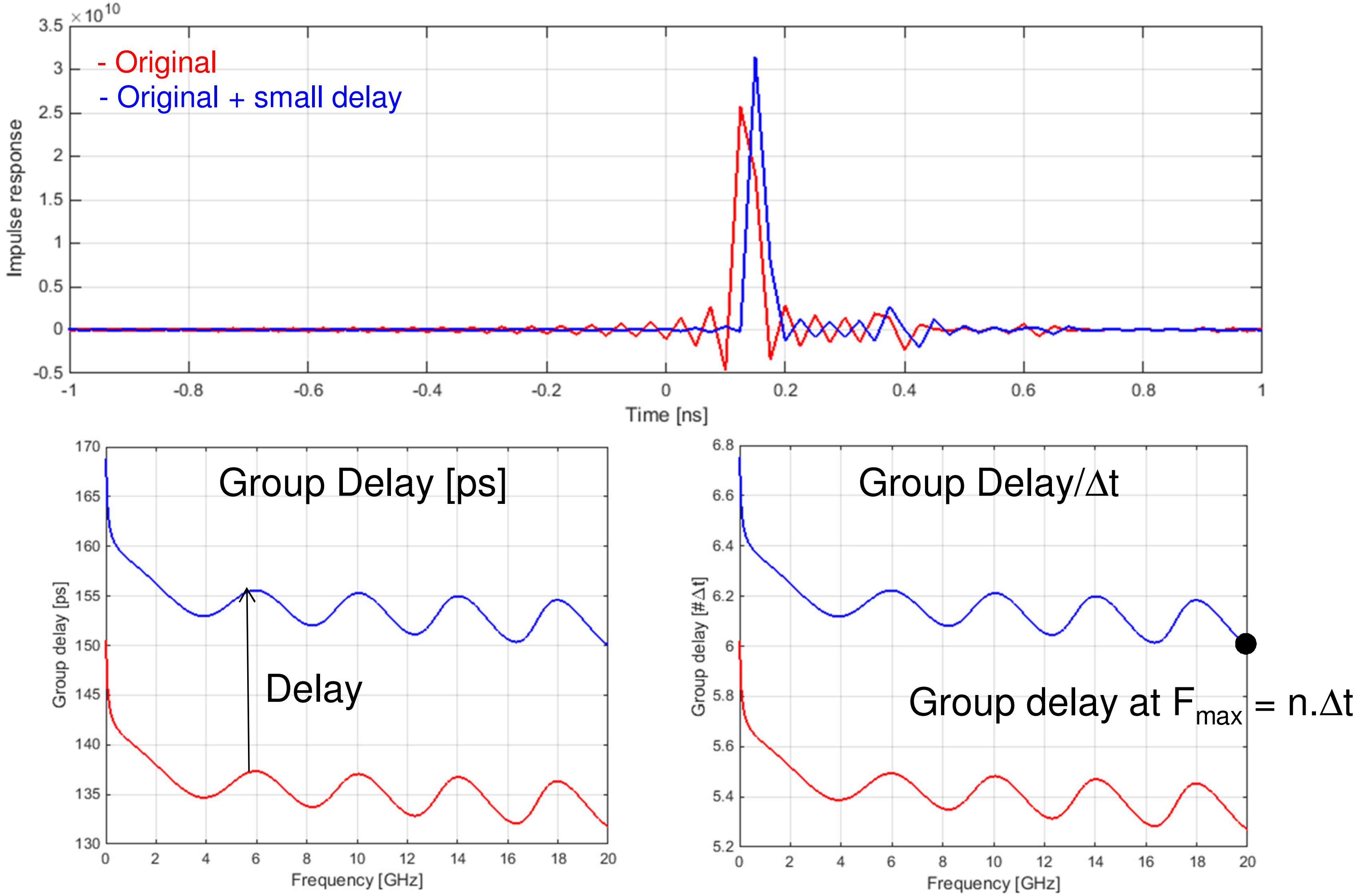


Techniques to minimize ringing - method 2



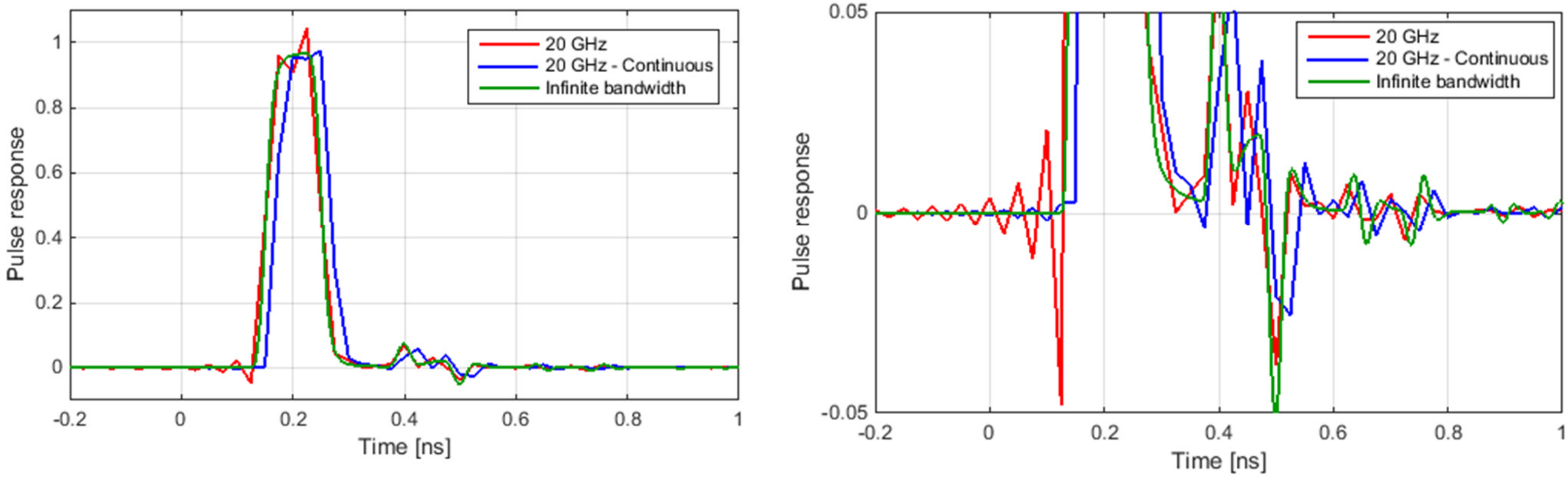


Techniques to minimize ringing - method 2



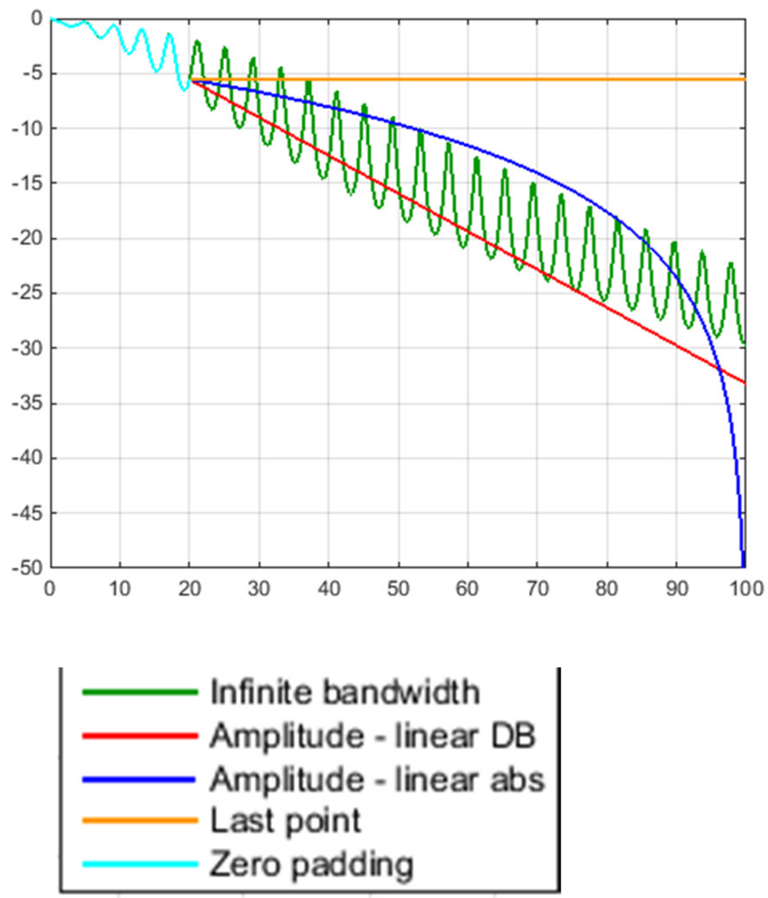
Techniques to minimize ringing - method 2

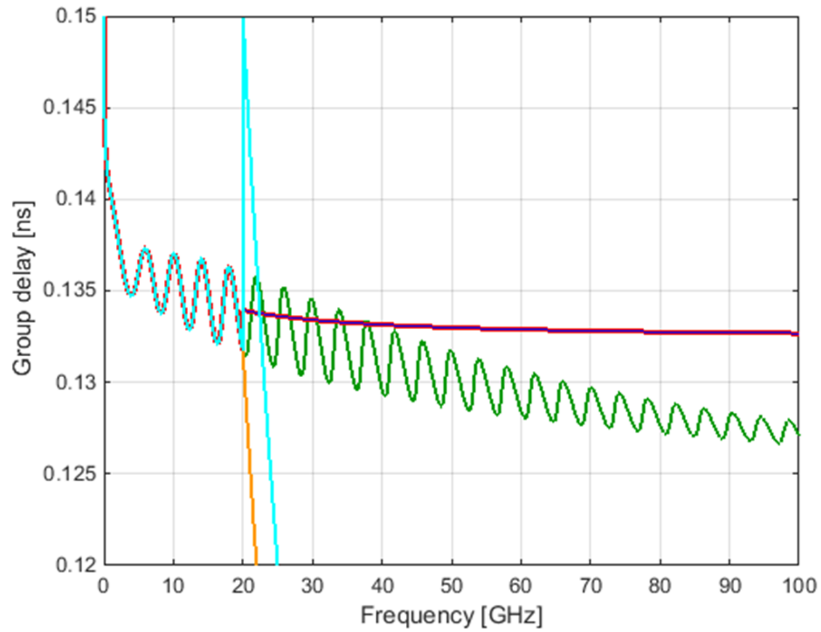
Pulse response



Techniques to reduce ringing - increase resolution

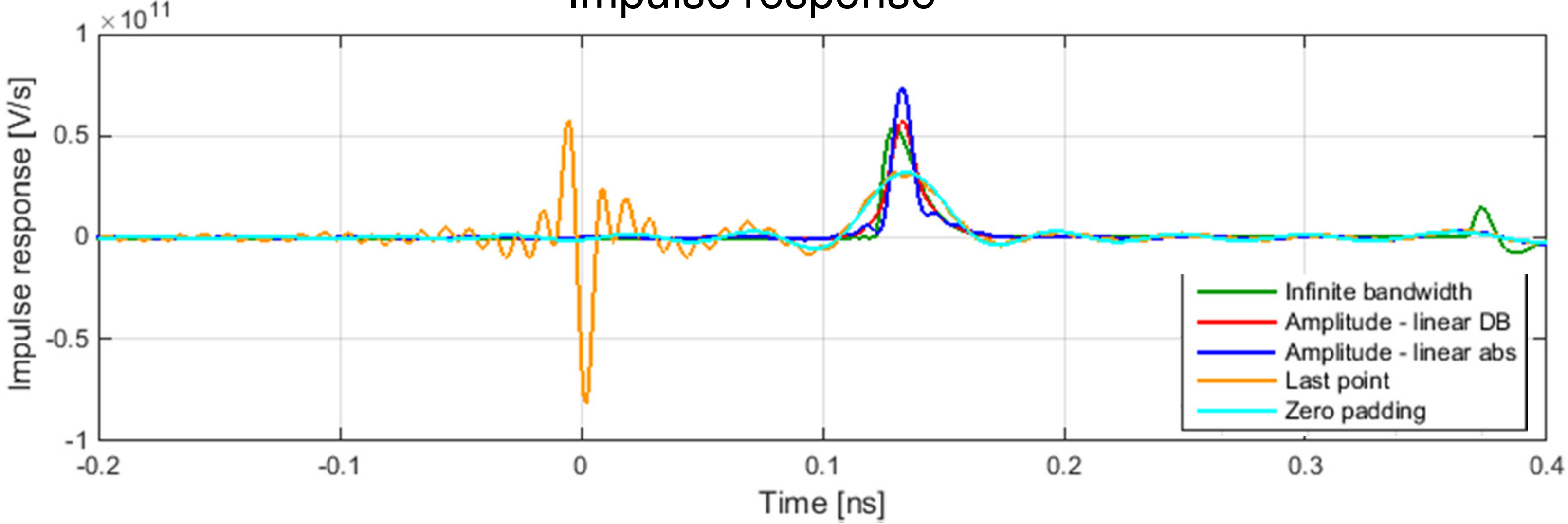
Extrapolation to 100 GHz



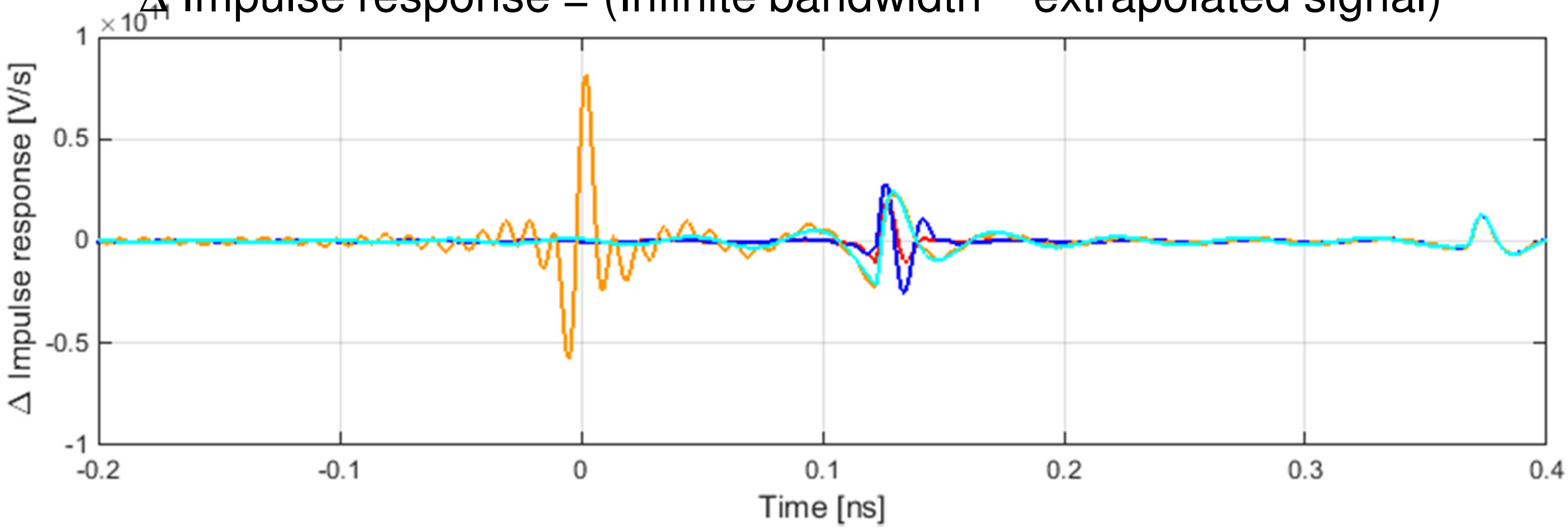


Techniques to reduce ringing - increase resolution

Impulse response

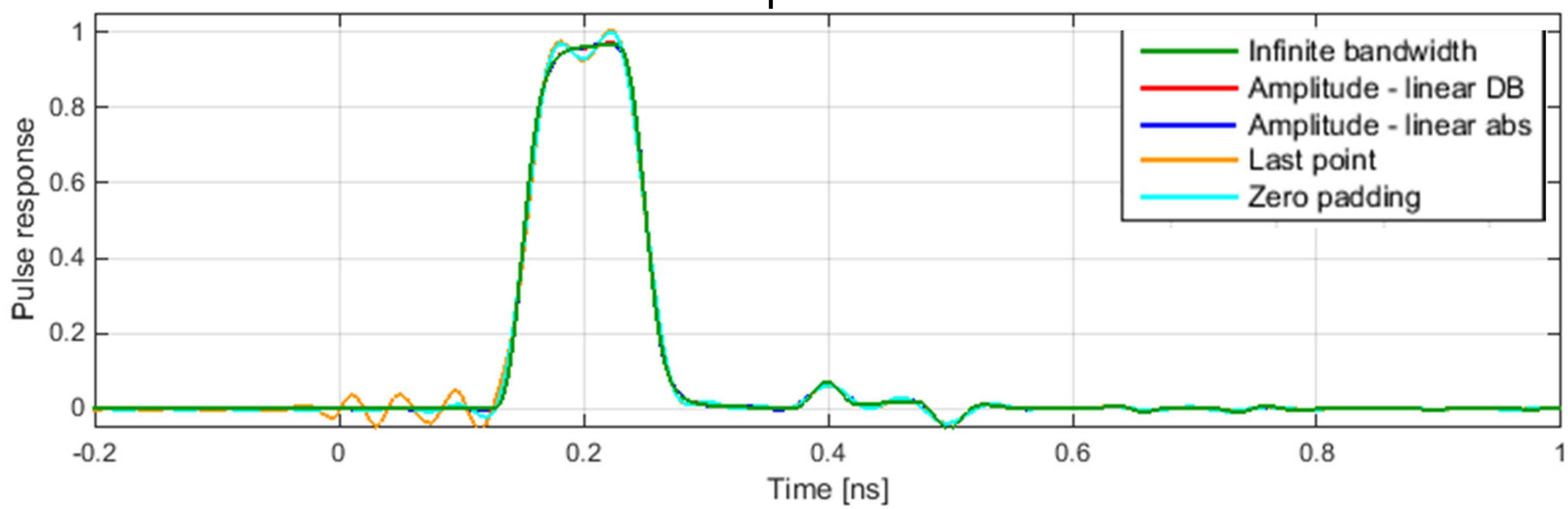


Δ Impulse response = (Infinite bandwidth – extrapolated signal)

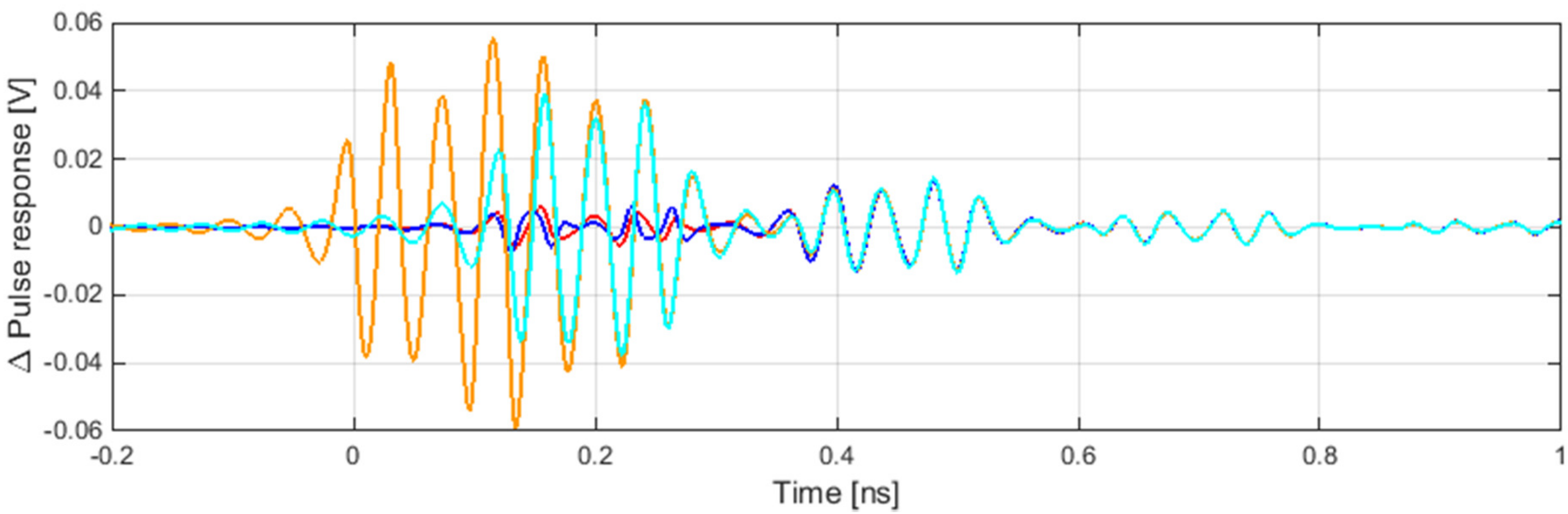


Techniques to reduce ringing - increase resolution

Pulse response

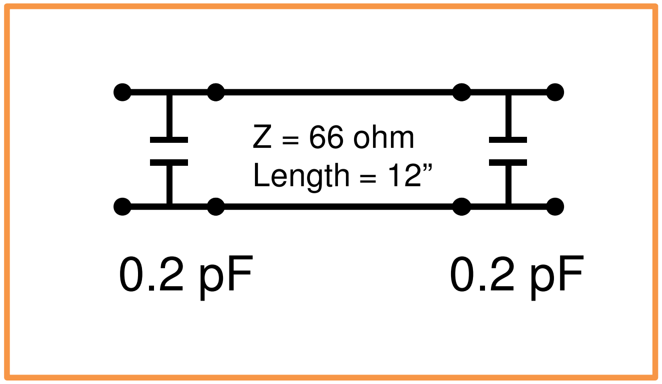


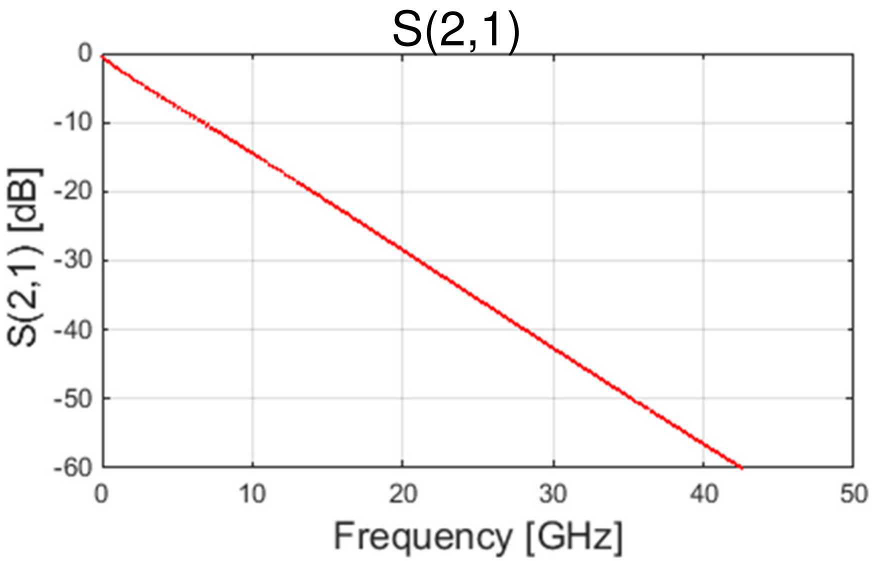
Δ Pulse response = Infinite bandwidth – extrapolated signal

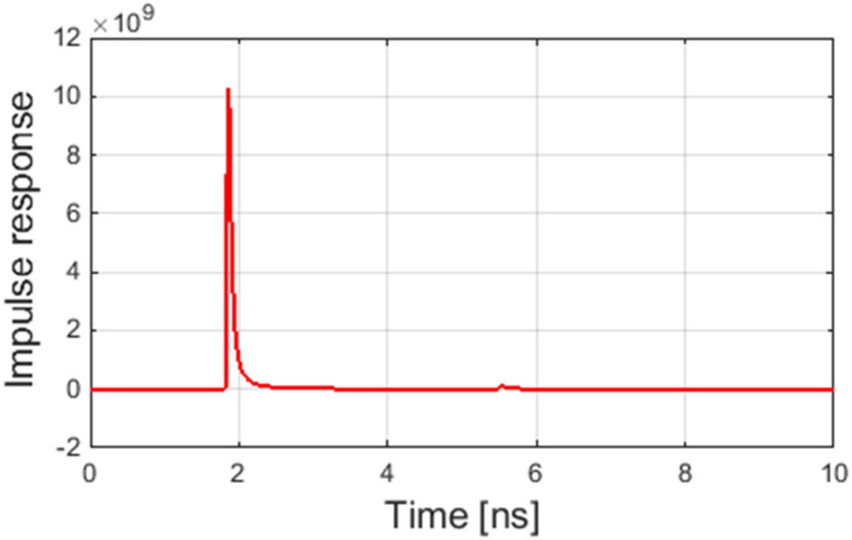


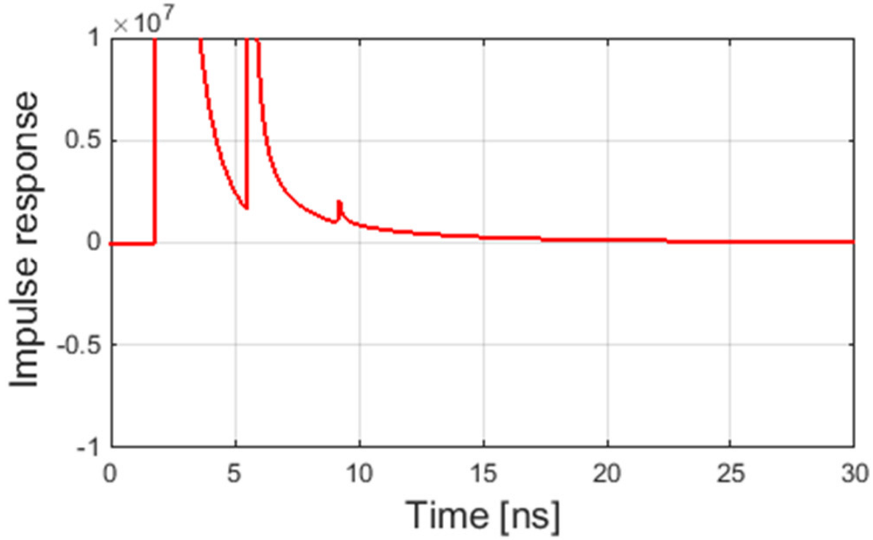
Causality issues due to discretization

Frequency domain resolution

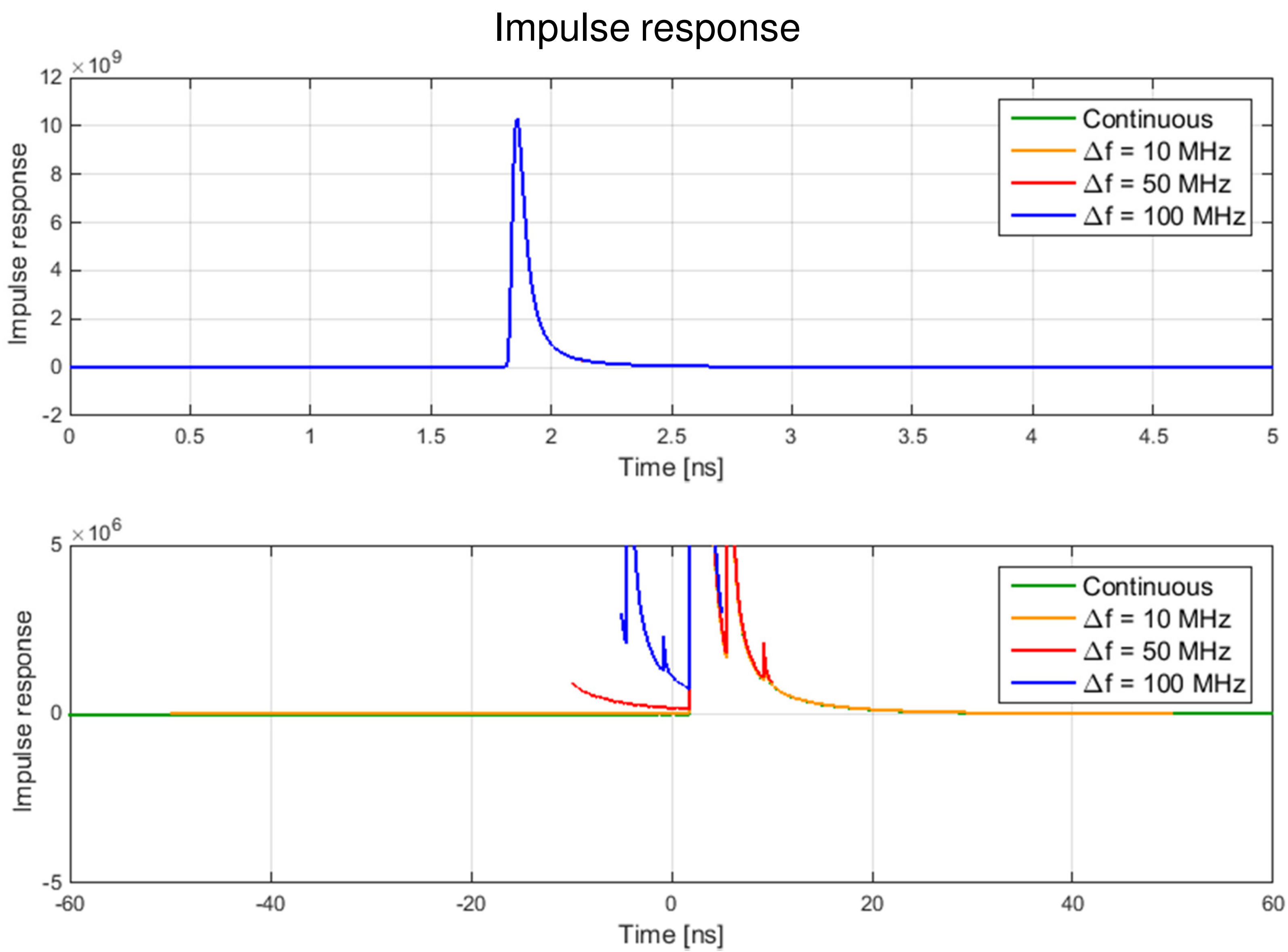






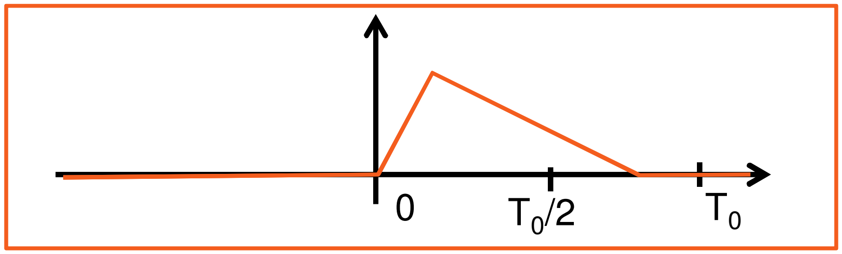


Frequency domain resolution

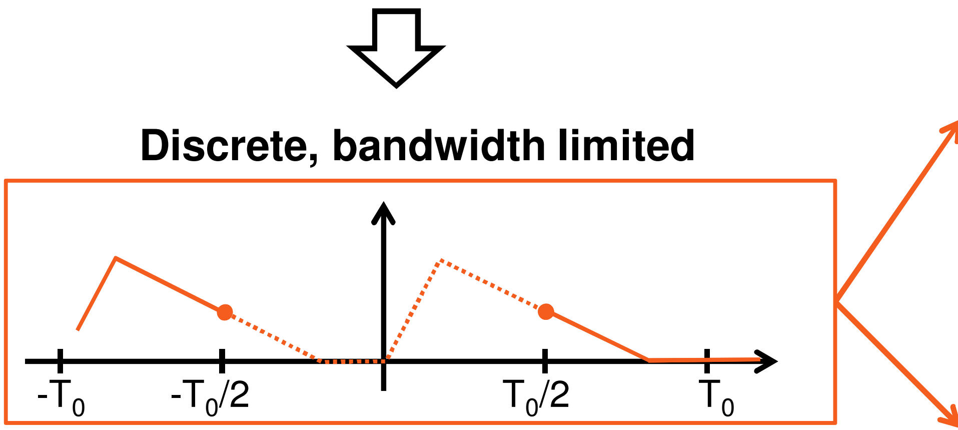


Frequency domain resolution

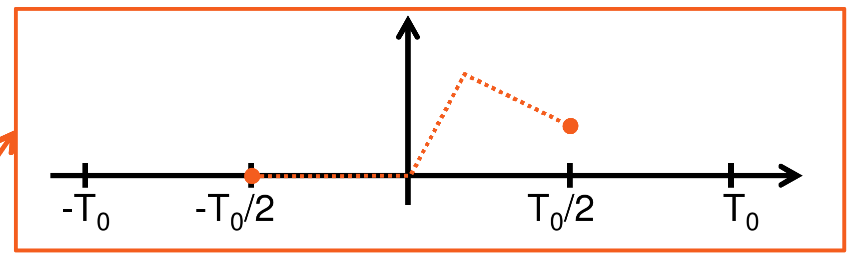
Continuous, infinite bandwidth



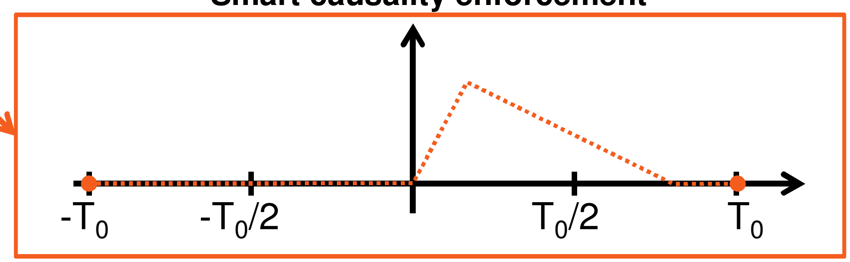
Discrete, bandwidth limited



Blind causality enforcement

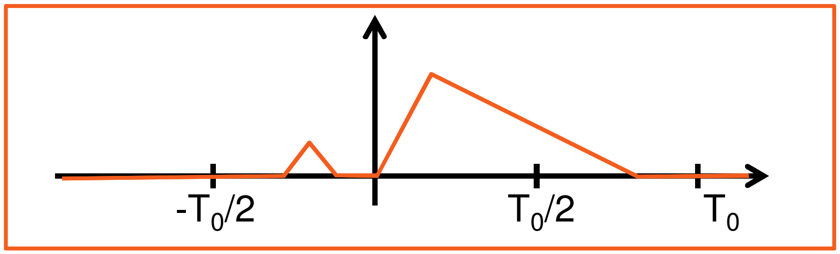


Smart causality enforcement

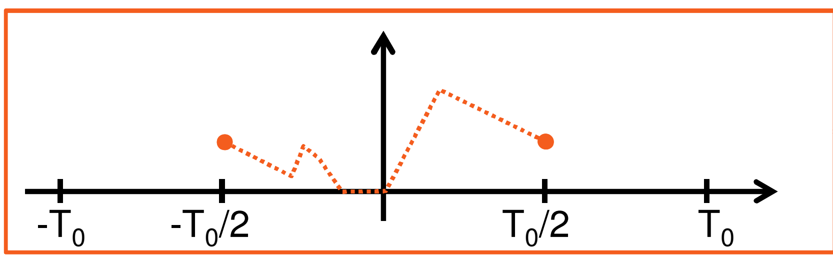


Frequency domain resolution

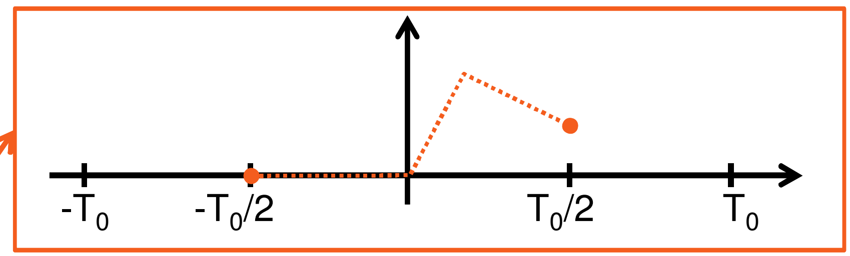
Continuous, infinite bandwidth



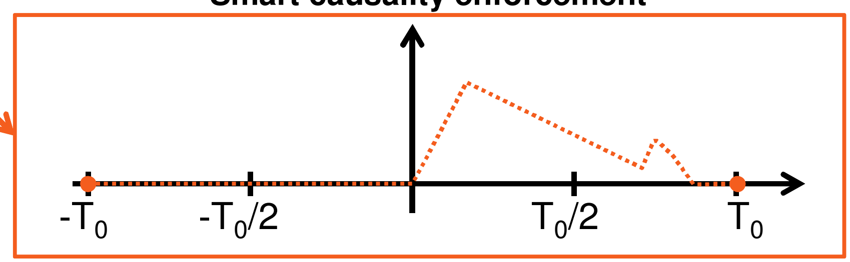
Discrete, bandwidth limited



Blind causality enforcement

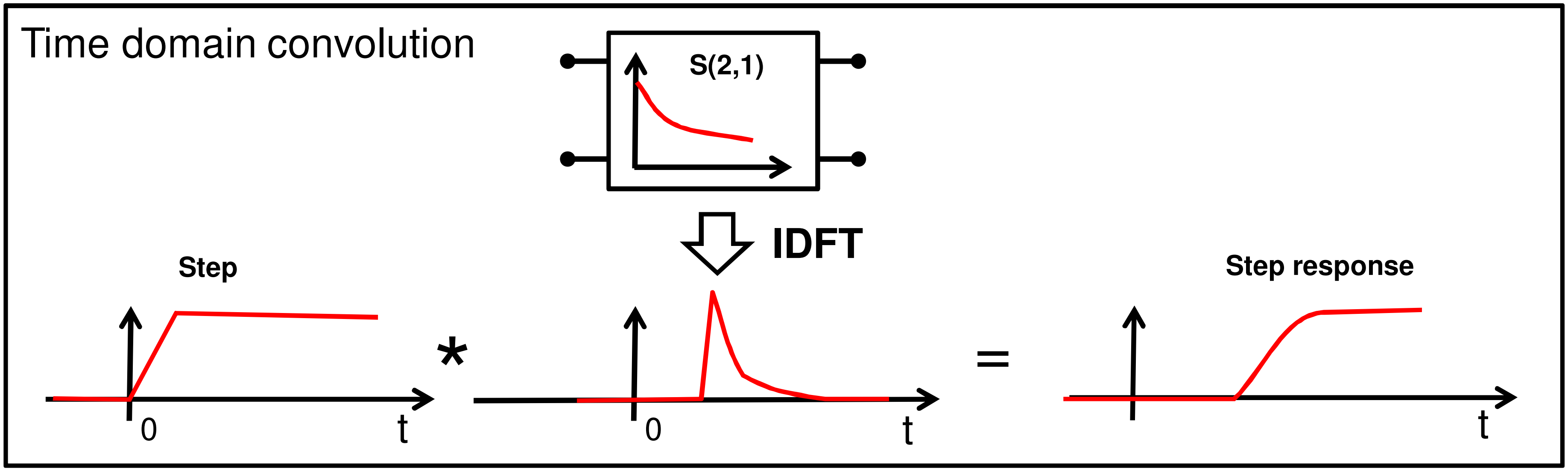


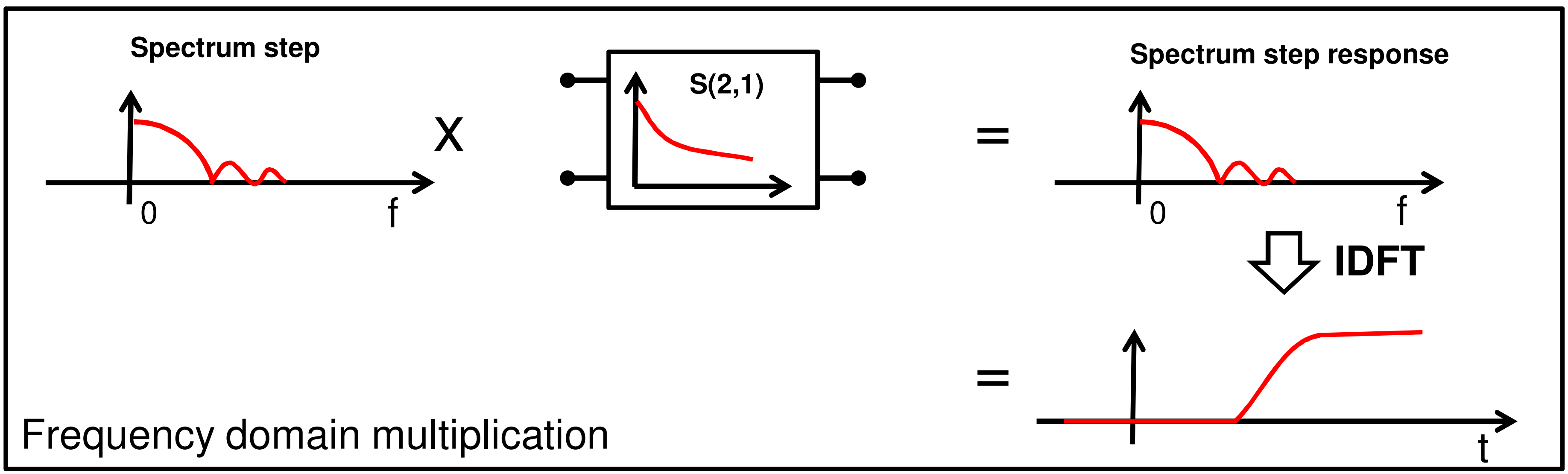
Smart causality enforcement



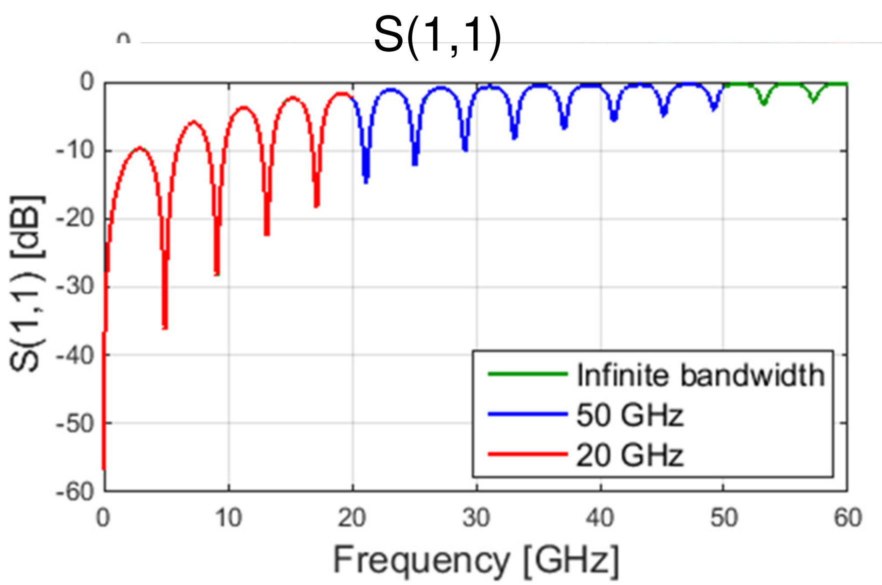
Step response

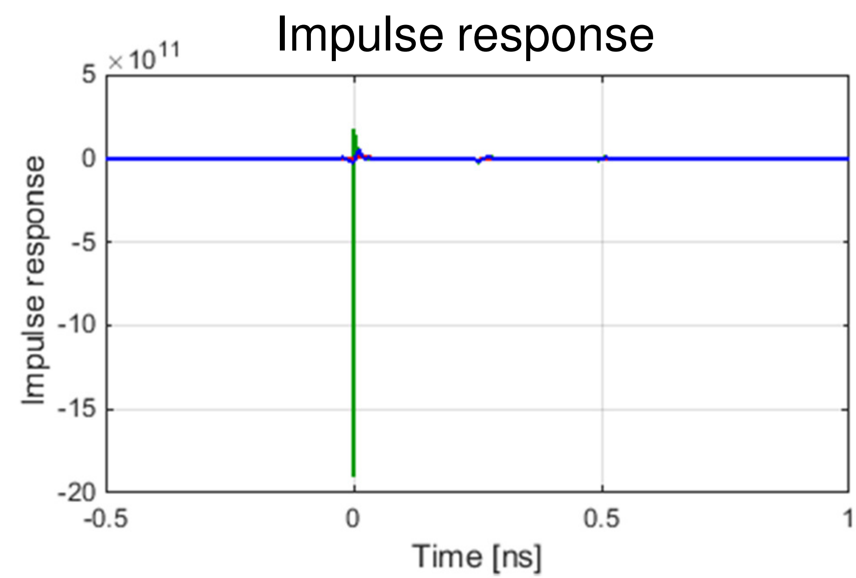
Calculation step response

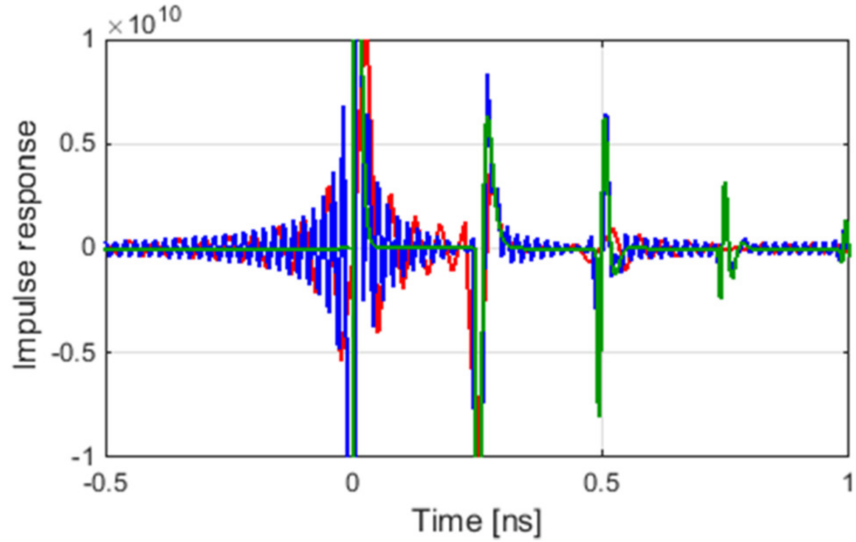




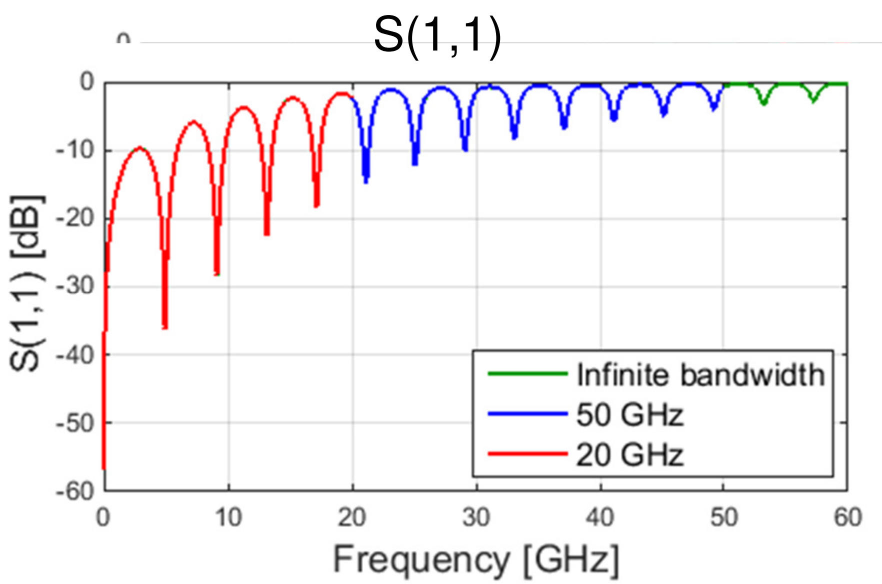
Calculation step response: example

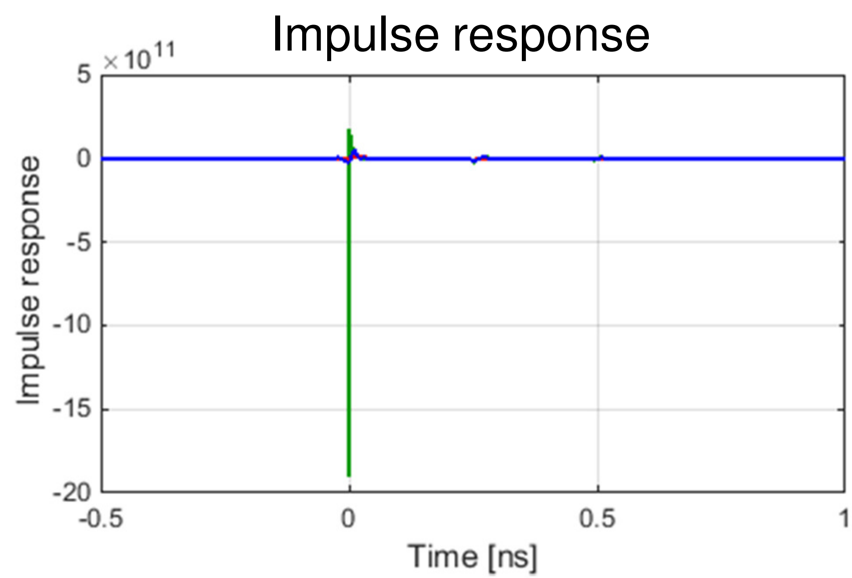


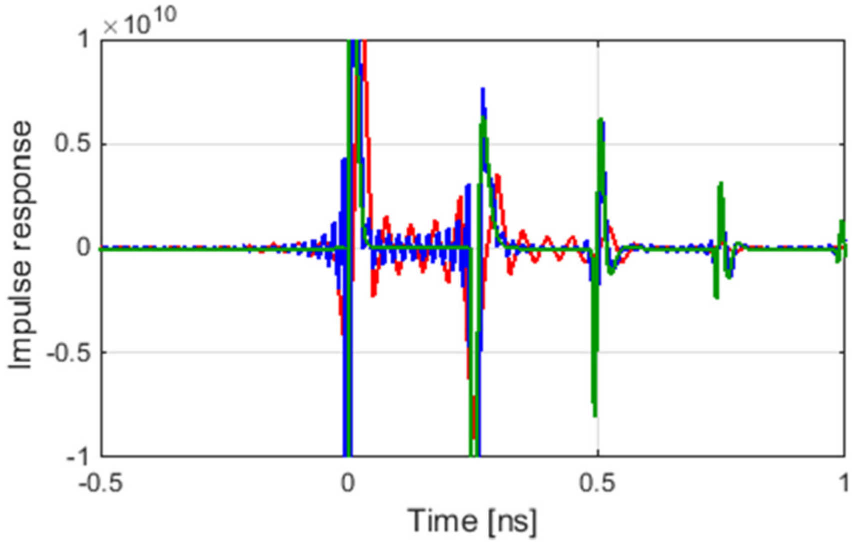




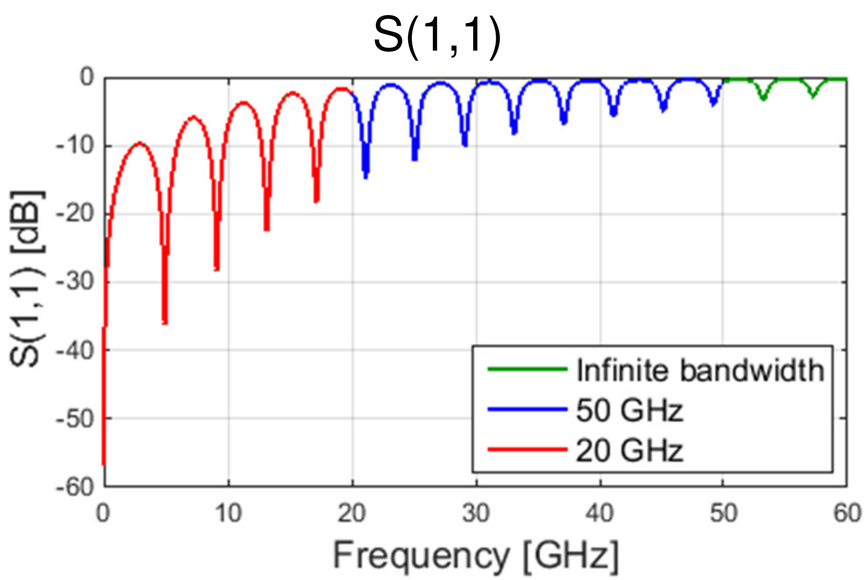
Calculation step response: example

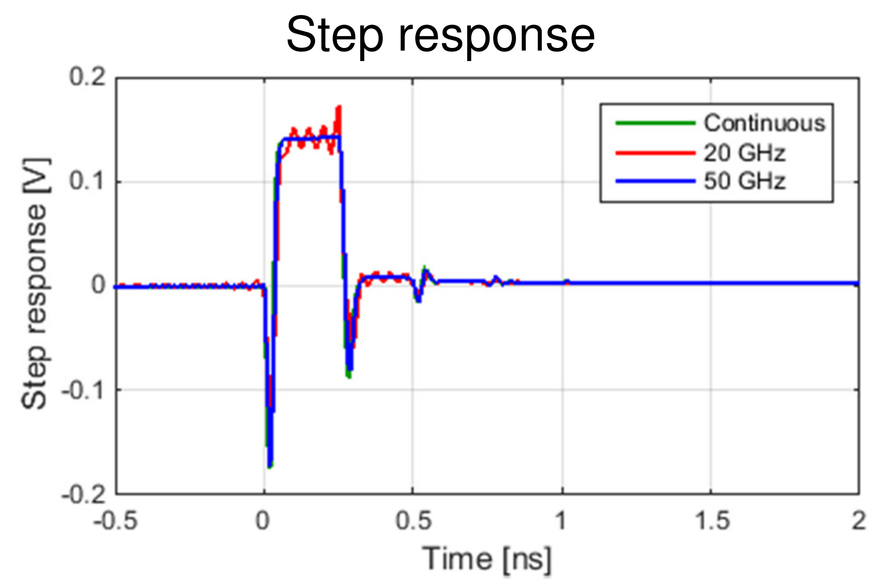


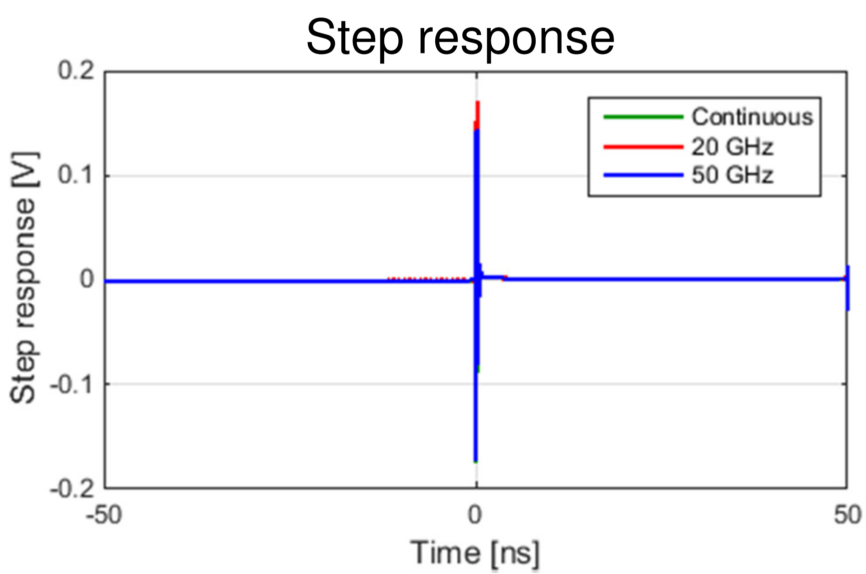


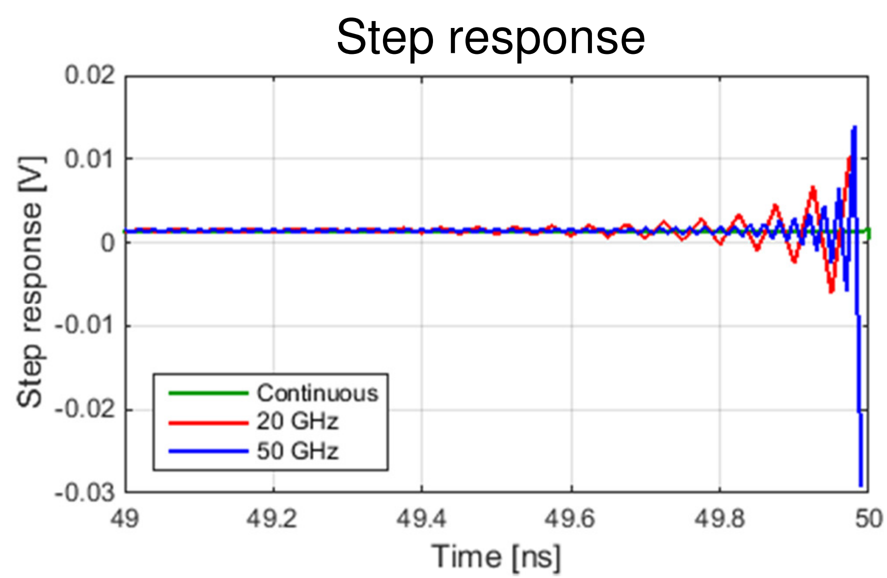


Calculation step response: example

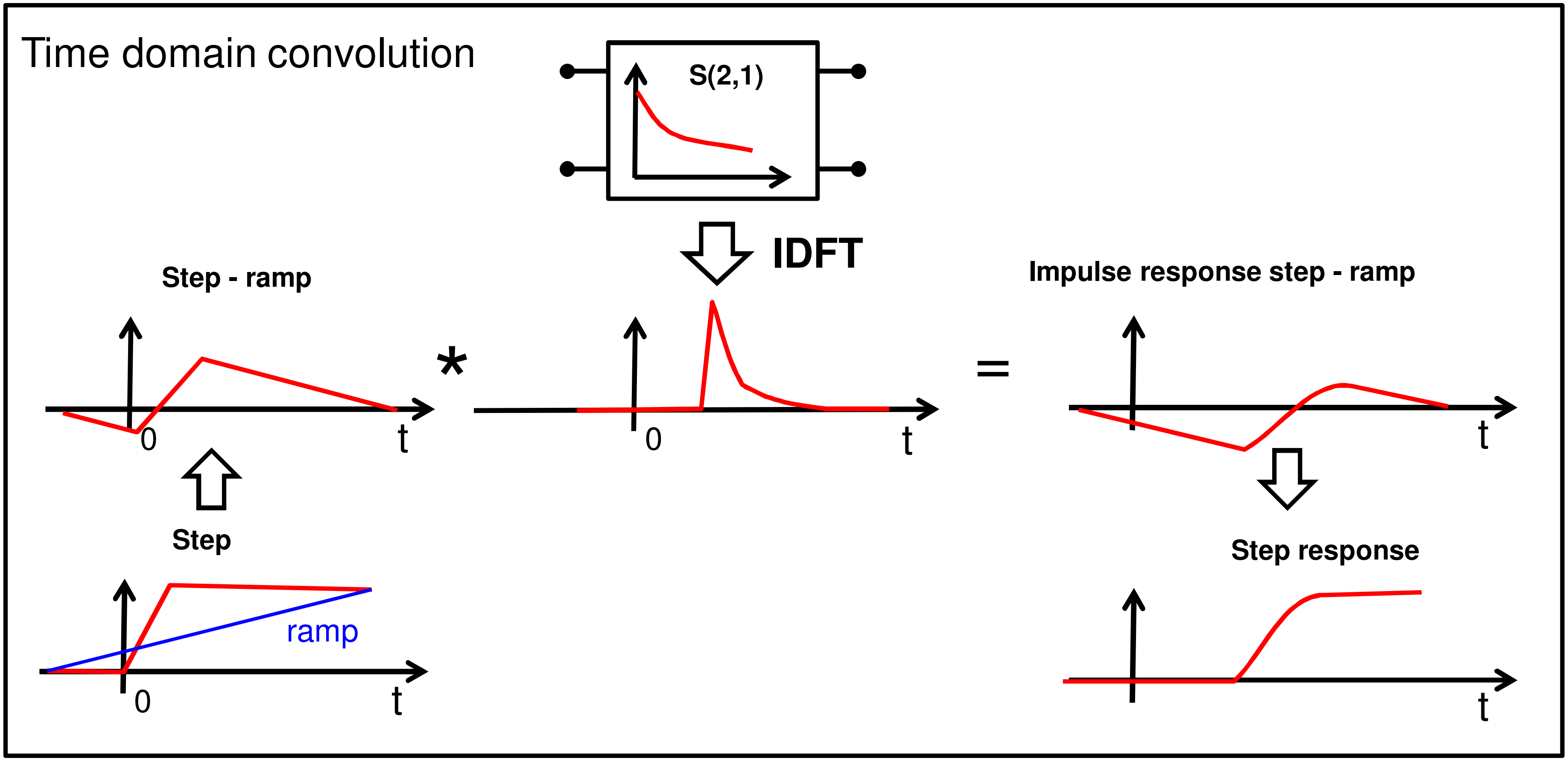








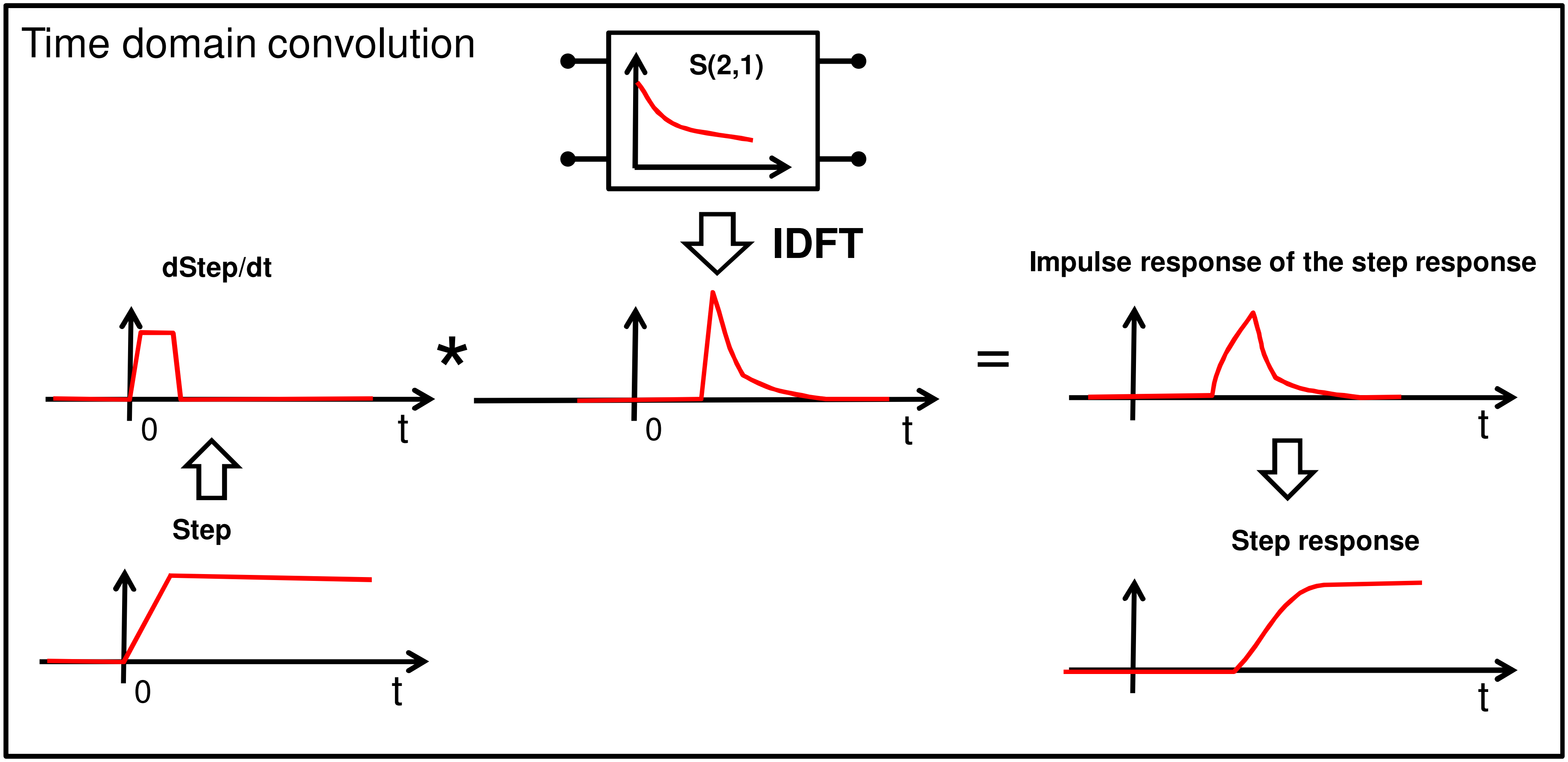
Calculation step response



Calculation step response

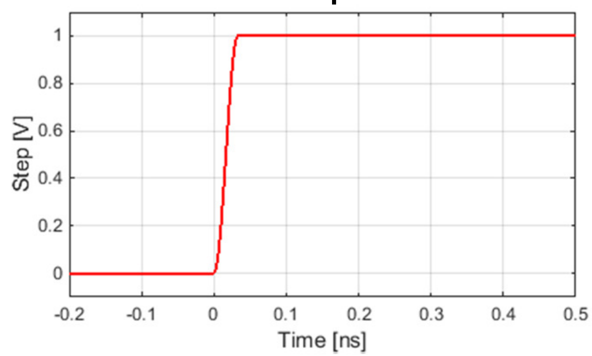
$$\text{step}(t) = \int_{-\infty}^{t} \text{impulse}(\tau).d\tau$$

$$\text{impulse}(t) = \frac{d\text{step}(t)}{dt}$$

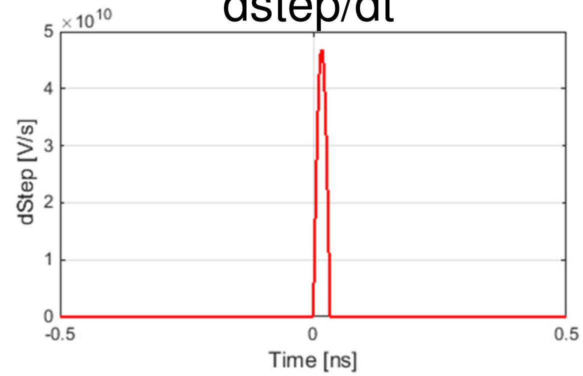


Calculation step response

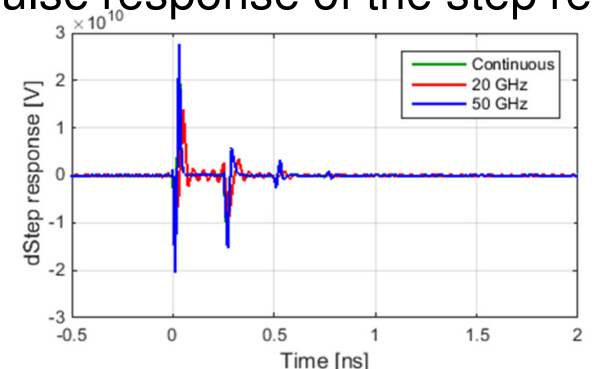
step



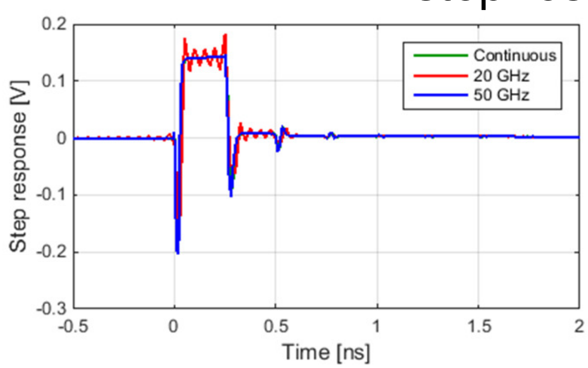
dstep/dt

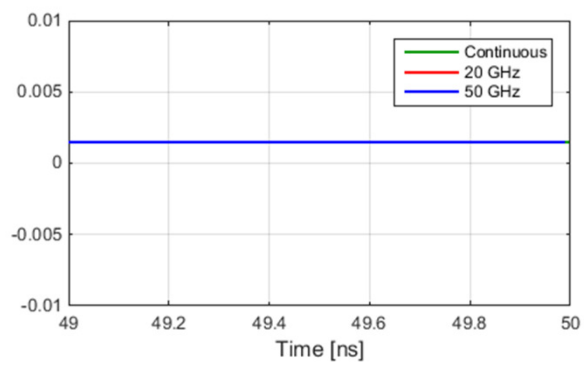


Impulse response of the step response



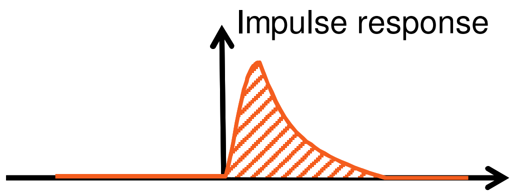
step response



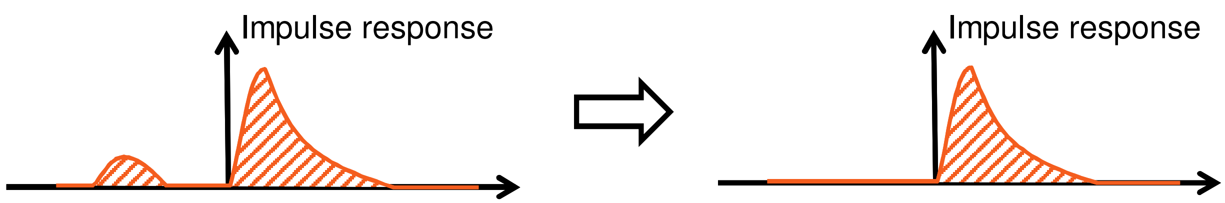


Impact on DC value when causality is enforced

DC value and causality enforcement



$$\mathrm{DC} = \int_{-\infty}^{+\infty} \text{impulse response}(t)\, dt$$

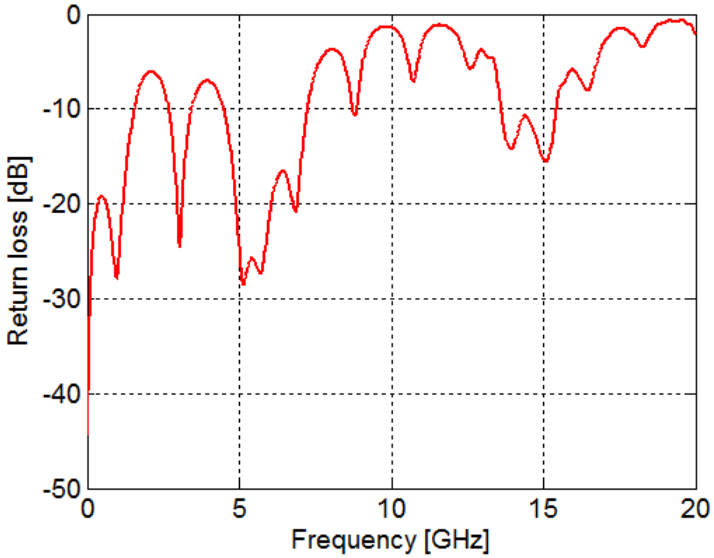


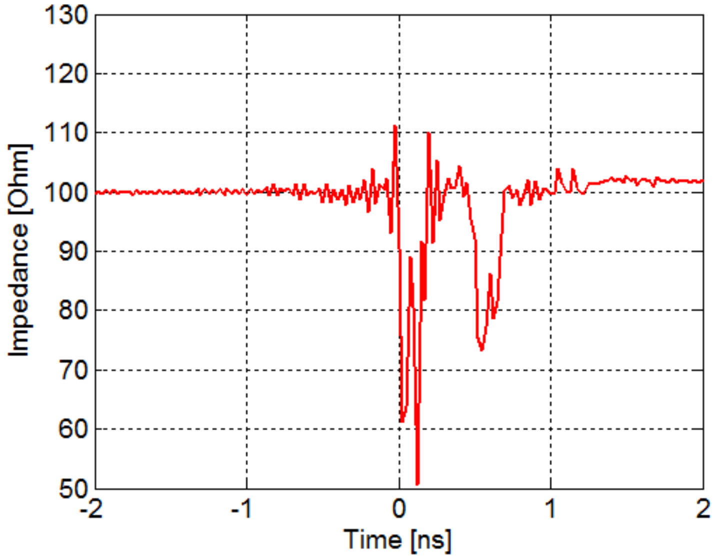
$$\text{new DC} = \int_{-\infty}^{+\infty} \text{impulse response}(t)\, dt - \int_{-\infty}^{0} \text{impulse response}(t)\, dt$$

DC-value changes if causality is enforced and $\int_{-\infty}^{0} \text{impulse response}(t)\, dt \neq 0$

DC value and causality enforcement

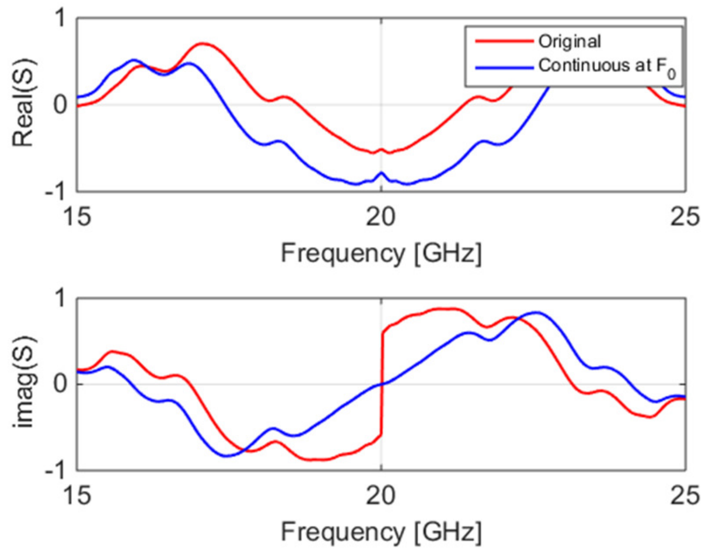
Measurement of connector and footprint: impedance is not-causal

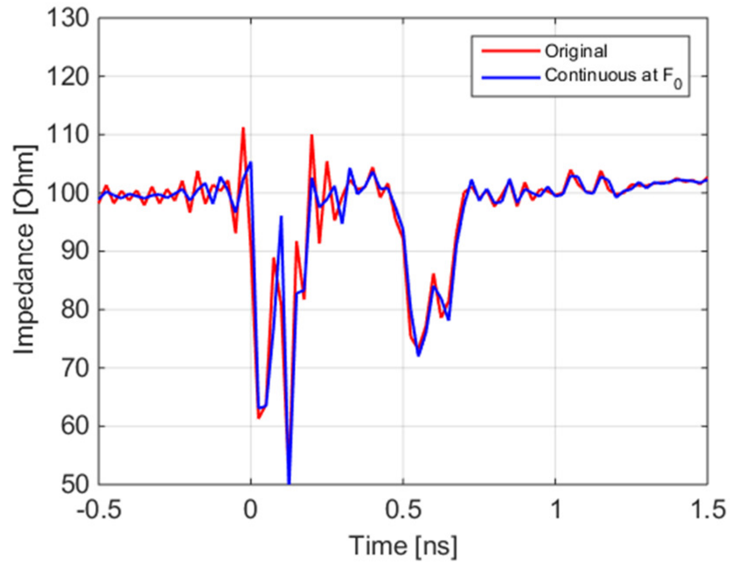




DC value and causality enforcement

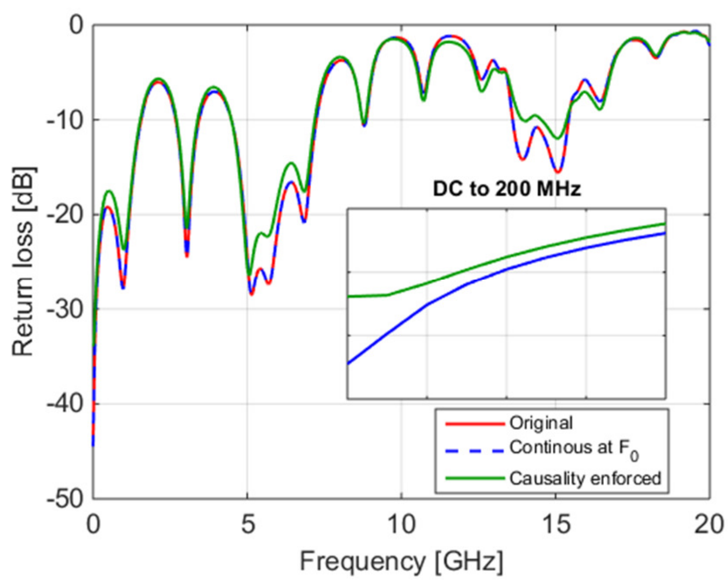
Step 1) Reduce ringing: make signal continuous at F_0:

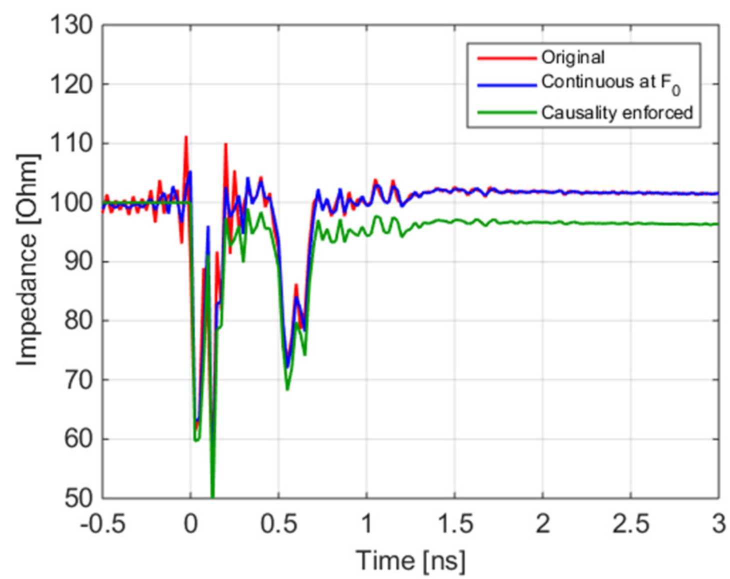




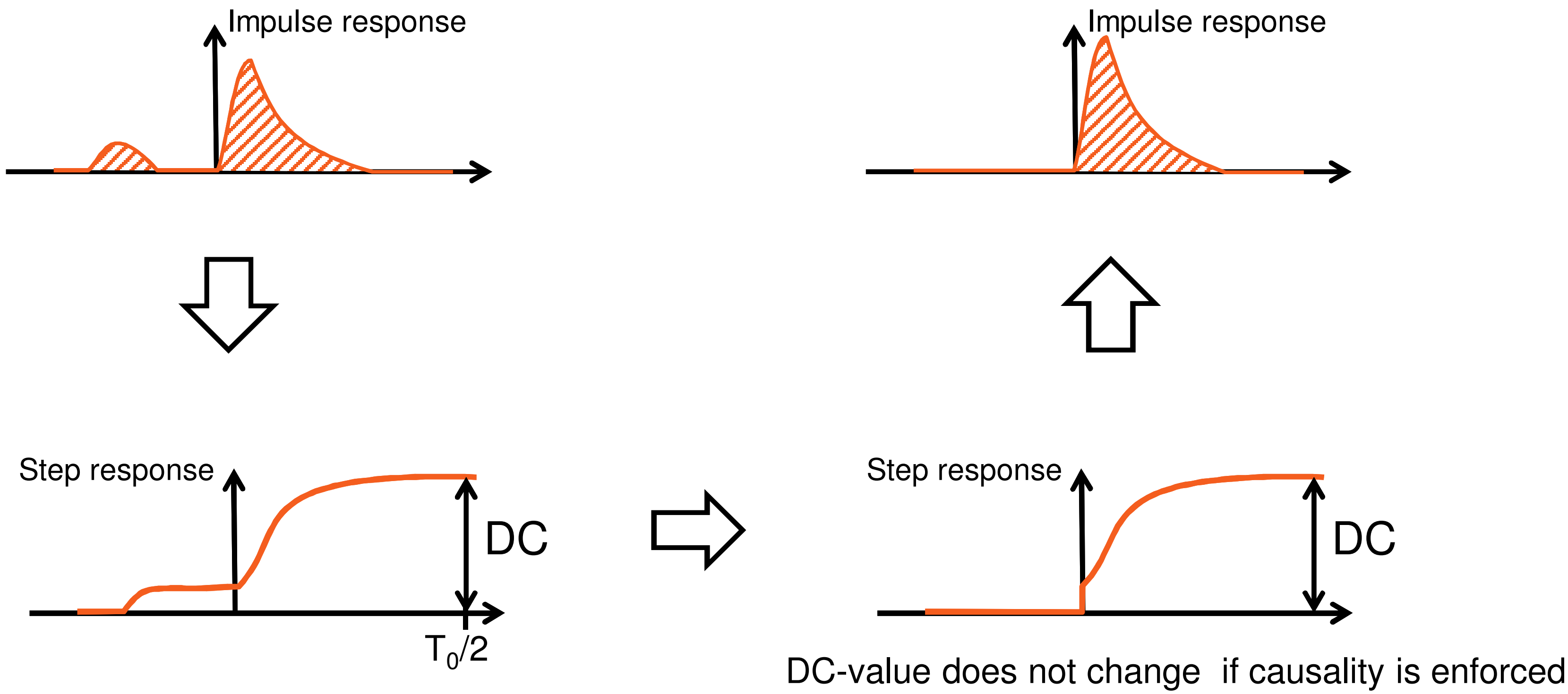
DC value and causality enforcement

Step 2) Enforce causality

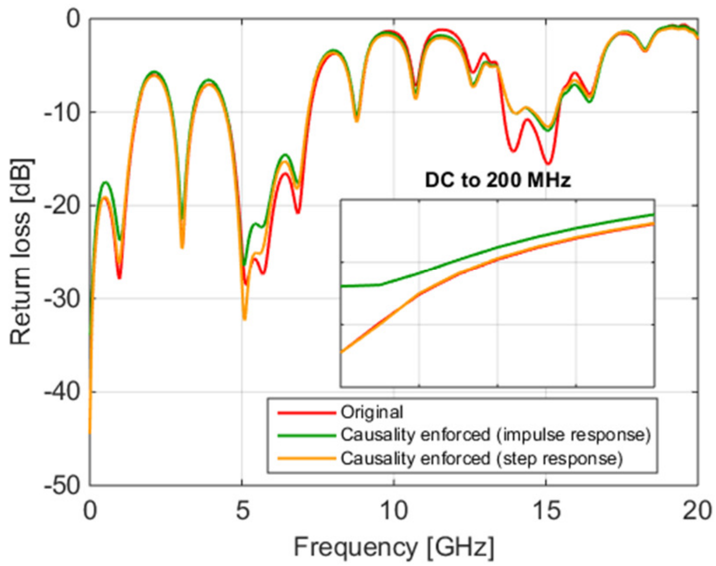


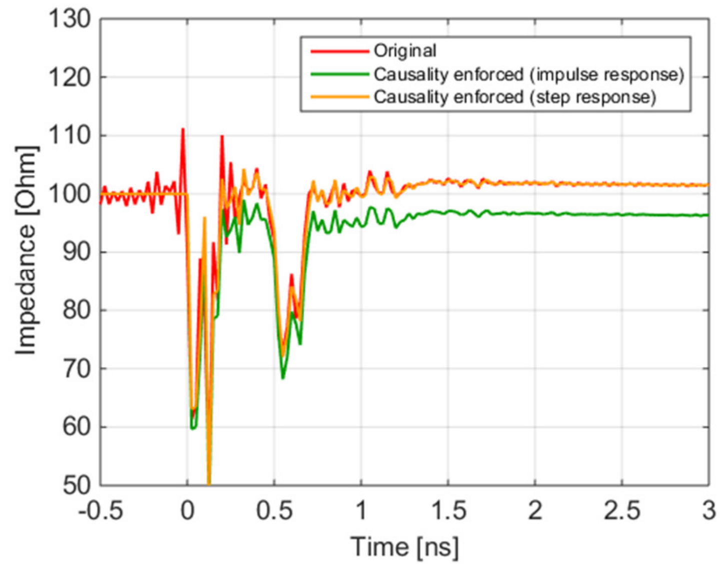


DC value and causality enforcement



DC value and causality enforcement






samtec