Analytic Solutions for Periodically Loaded Transmission Line Modeling

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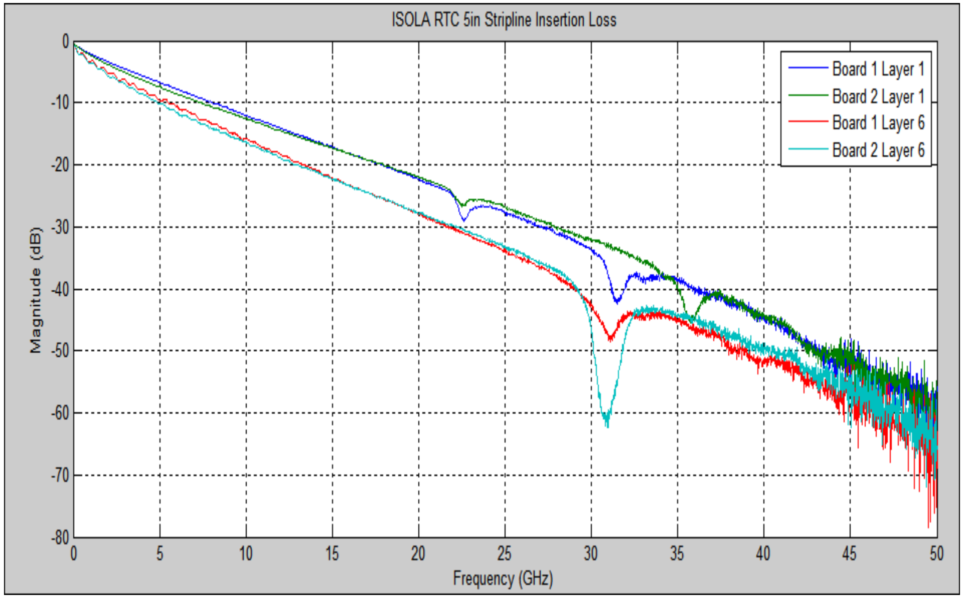
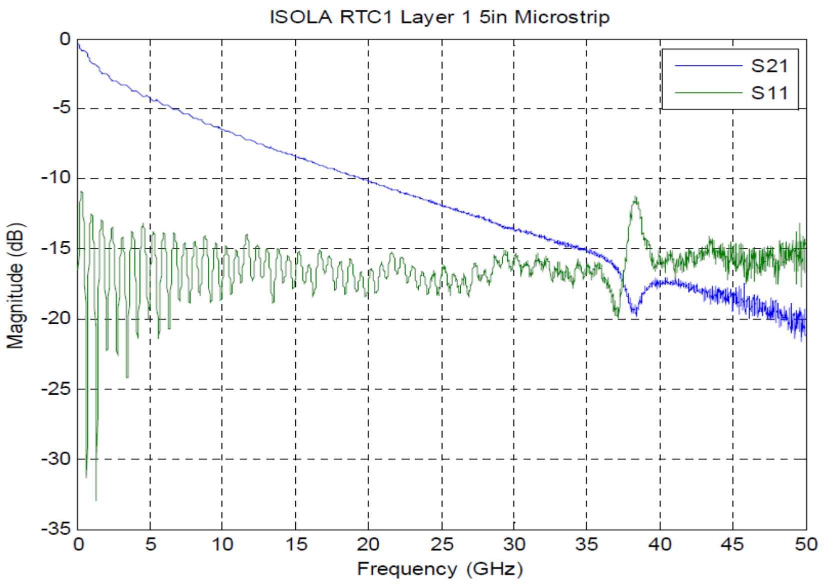
Outline

1. Motivation.
    a. Measurement Observations
    b. Periodic Structures in Digital Signal Interconnects
    c. Periodic Fiber Weave Effect
2. Theory of Wave Propagation in periodic medium.
    a. Solving Wave Equations
    b. Simulation Examples
    c. Transmission Line Analogy
    d. Cascading Periodic Structures- Chebyshev Identity
    e. Simulation Examples
3. Applications of theory to Signal Integrity problems.
    a. Simulation Examples
4. Periodic Loading impact on system eye margins.
5. Summary.

Measurement Observations





- Resonances observed in high frequency measurements on Printed Circuit Boards.
- These resonances could degrade the Signal Integrity of system buses.
- PCP properties including fiber weaves were suspected as cause, but not conclusive.

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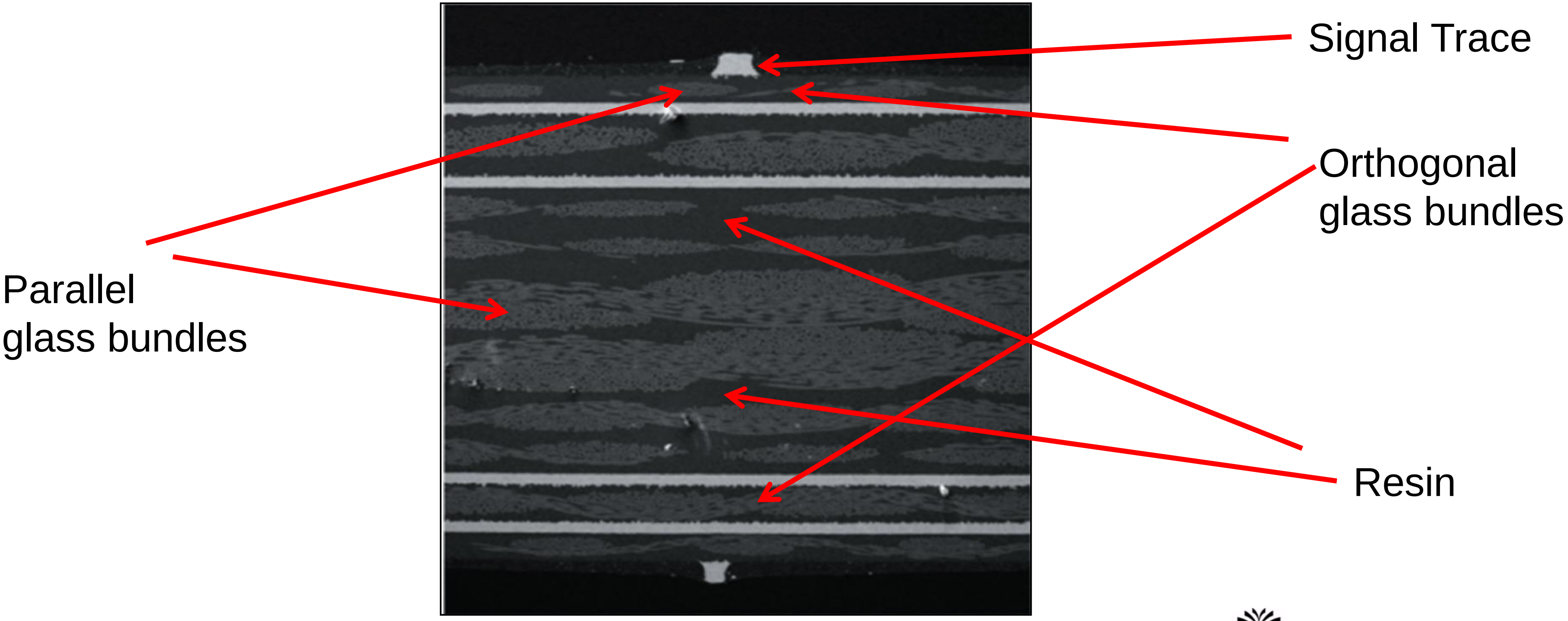
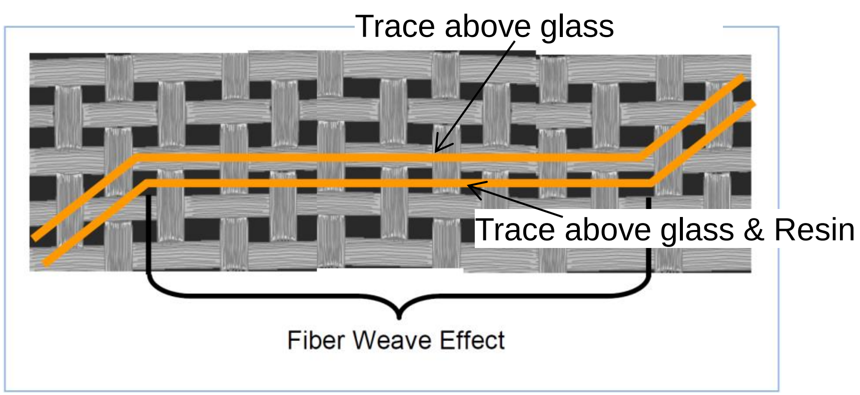
Periodic Fiber Weave Effect

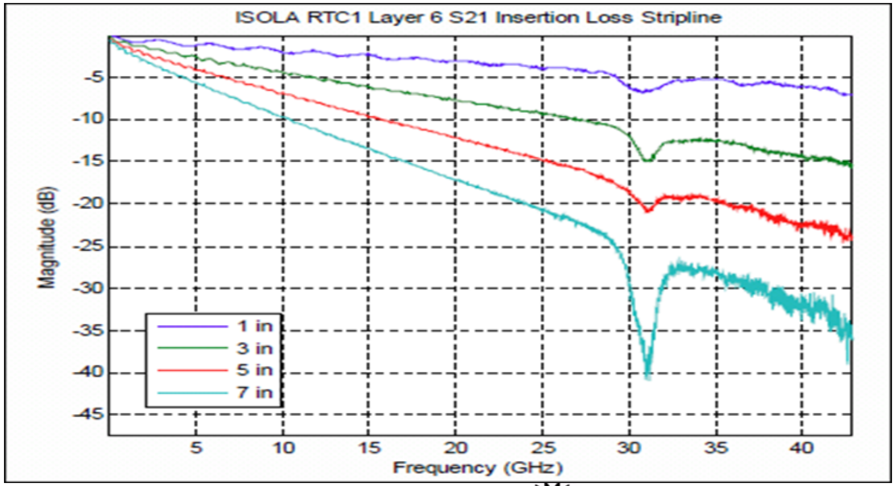
- PCBs are composed of resin and a woven glass fabric reinforcement
- Different electrical properties of two materials introduce a periodic medium for propagating waves .



Periodic Fiber Weave Effect

- Differential signaling skew could be introduced by one trace laying above resin region and other on glass region.
- Additional power losses due to periodically altering dielectric constant and loss tangent could degrade the Signal Integrity of the System.





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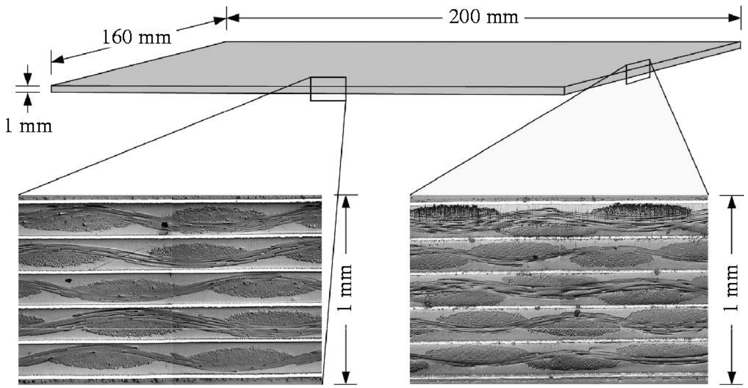
Periodic Structures in High Speed interconnects

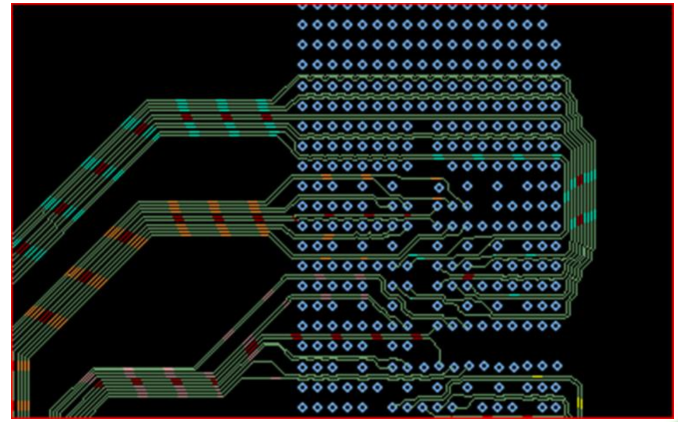
Fiber Weaves in PCBs



(a) Warp yarns parallel to the page, fill yarns perpendicular

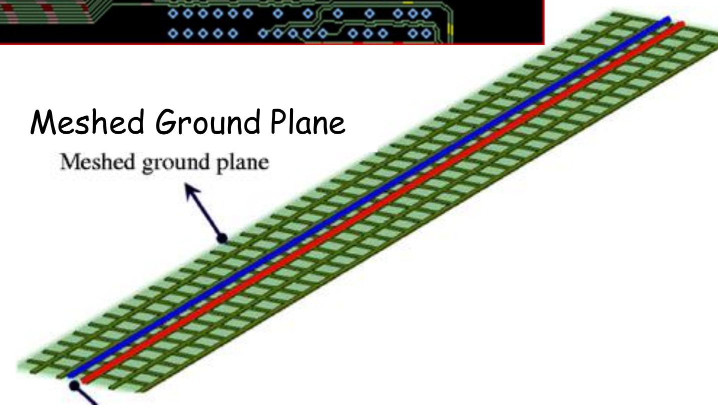
(b) Fill yarns parallel to the page, warp yarns perpendicular

Routings in BGA field

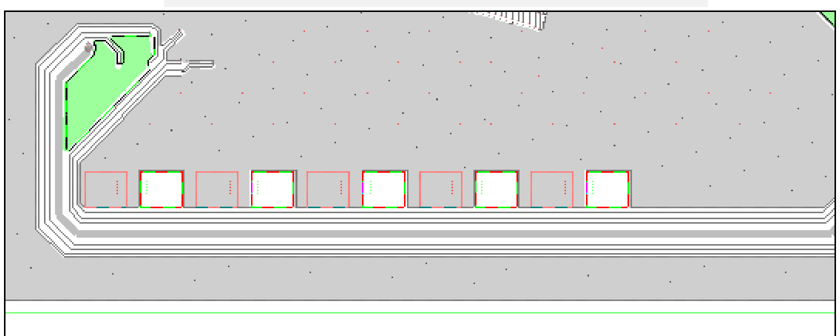


Braided Cable Shielding

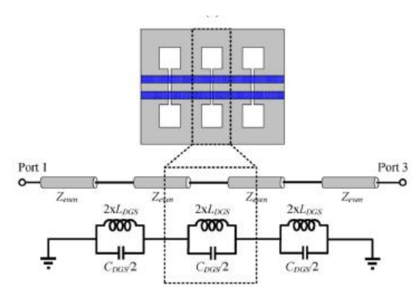
Meshed Ground Plane



Component placement on PCBs



Embedded Common Mode Chokes



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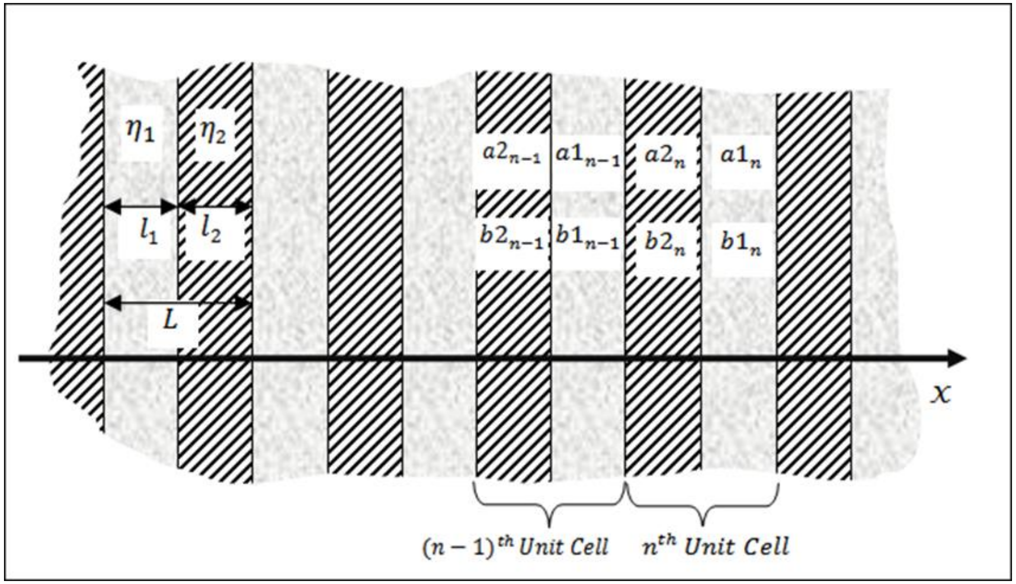
Periodic Wave Propagation Theory (method #1)



Consider an infinitely long one dimensional two tone medium, composed of periodically varying dielectric properties.

$$\eta(x) = \begin{cases} \eta_1 = \sqrt{\varepsilon_{r1}\mu_{r1}}, & 0 < x < l_1 \\ \eta_2 = \sqrt{\varepsilon_{r2}\mu_{r2}}, & l_1 < x < l_1 + l_2 \end{cases}$$

$$\eta(x+L) = \eta(x)$$

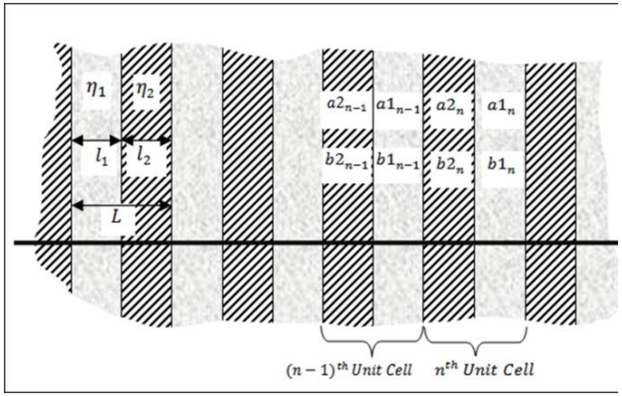
Define a_n and b_n as Complex amplitudes of the incident and reflected modal fields in medium 1 and medium 2 in n^{th} unit cell,

These vector fields can be in column matrices

$$\begin{pmatrix} a1_n \\ b1_n \end{pmatrix} - Medium\ 1\ of\ Unit\ cell\ n$$

$$\begin{pmatrix} a2_n \\ b2_n \end{pmatrix} - Medium\ 2\ of\ Unit\ cell\ n$$

Periodic Wave Propagation Theory (method #1)



Wave equation for propagating E (or H) field :

$$E(x,z) = E(x)e^{j\beta z}$$

Electric field intensity distributions in these regions can be expressed as

$$E1(x,z) = \left\{a1_n e^{jk_1(x-nL)} + b1_n e^{-jk_1(x-nL)}\right\} e^{j\beta z}$$

$$E2(x,z) = \left\{a2_n e^{jk_2(x-nL)} + b2_n e^{-jk_2(x-nL)}\right\} e^{j\beta z}$$

complex wave numbers and can be expressed as

$$k_1 = \sqrt{\left\{\left[\left(\frac{\omega}{c}\right)\eta_1\right]^2 - \beta^2\right\}} \qquad k_2 = \sqrt{\left\{\left[\left(\frac{\omega}{c}\right)\eta_2\right]^2 - \beta^2\right\}}$$

Periodic Boundary Condition

E vector and its derivative $\partial E/\partial x$ are continuous at any arbitrary interface in y-z plane.

Continuity on E(x) ➔

$$a1_{n-1} + b1_{n-1} = e^{-jk_2L}a2_n + e^{jk_2L}b2_n$$

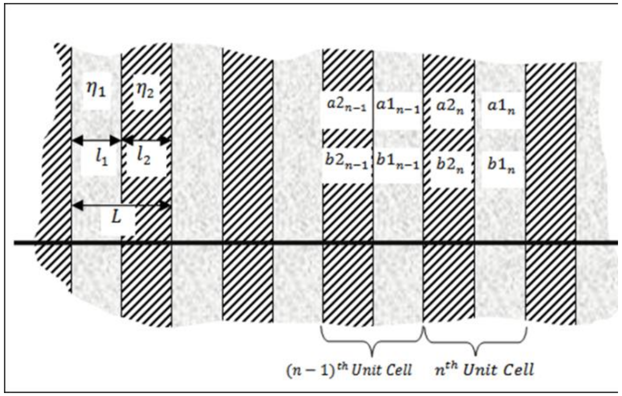
$$e^{-jk_2l_1}a2_n + e^{jk_2l_1}b2_n = e^{-jk_1l_1}a1_n + e^{jk_1l_1}b1_n$$

Continuity on ∂E/(∂x) ➔

$$k_1\left(a1_{n-1} - b1_{n-1}\right) = k_2\left(e^{jL}a2_n - e^{jk_2L}b2_n\right)$$

$$k_2\left(e^{-jk_2l_1}a2_n - e^{jk_2l_1}b2_n\right) = k_1\left(e^{jk_1l_1}a1_n - e^{jk_1l_1}b1_n\right)$$

Periodic Wave Propagation Theory (method #1)



Periodic Boundary Condition(Contd..)

Solve continuity condition equations for a_n and b_n waves of TE modes (eliminate a2 & b2):

$$\begin{pmatrix} a1_{n-1} \\ b1_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a1_n \\ b1_n \end{pmatrix}$$

This is the classical definition of "ABCD" parameters

For TE modes:

$$A = e^{-jk_1 l_1}\left[\cos k_2 l_2 - \frac{1}{2} j \left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right) \sin k_2 l_2\right]$$

$$B = e^{jk_1 l_1}\left[-\frac{1}{2} j \left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right) \sin k_2 l_2\right]$$

$$C = e^{-jk_1 l_1}\left[\frac{1}{2} j \left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right) \sin k_2 l_2\right]$$

$$D = e^{jk_1 l_1}\left[\cos k_2 l_2 + \frac{1}{2} j \left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right) \sin k_2 l_2\right]$$

For TM modes:

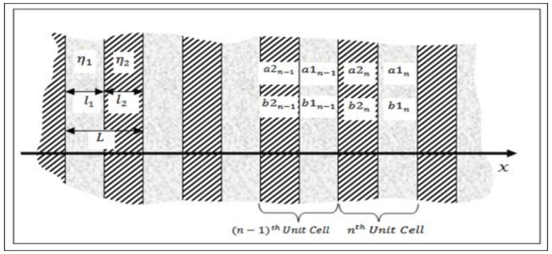
$$A' = e^{-jk_1 l_1}\left[\cos k_2 l_2 - \frac{1}{2} j \left(\frac{\eta_2^2 k_1}{\eta_1^2 k_2} + \frac{\eta_1^2 k_2}{\eta_2^2 k_1}\right) \sin k_2 l_2\right]$$

$$B' = e^{jk_1 l_1}\left[-\frac{1}{2} j \left(\frac{\eta_2^2 k_1}{\eta_1^2 k_2} - \frac{\eta_1^2 k_2}{\eta_2^2 k_1}\right) \sin k_2 l_2\right]$$

$$C' = e^{-jk_1 l_1}\left[\frac{1}{2} j \left(\frac{\eta_2^2 k_1}{\eta_1^2 k_2} - \frac{\eta_1^2 k_2}{\eta_2^2 k_1}\right) \sin k_2 l_2\right]$$

$$D' = e^{jk_1 l_1}\left[\cos k_2 l_2 + \frac{1}{2} j \left(\frac{\eta_2^2 k_1}{\eta_1^2 k_2} + \frac{\eta_1^2 k_2}{\eta_2^2 k_1}\right) \sin k_2 l_2\right]$$

Periodic Wave propagation Theory



$$\begin{pmatrix} a1_{n-1} \\ b1_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a1_n \\ b1_n \end{pmatrix}$$

Assume a Reciprocal medium ⇨

$$AD - BC = 1$$

Floquet's Theory for periodic wave propagation ⇨

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{-jKL} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

Where, e^{-jKL} is the eigenvalue of ABCD matrix ⇨

$$e^{-2jKL} - (A + D)e^{-jKL} + AD - BC = 0$$

⇨ dispersion relationship

$$K(\beta, \omega) = \frac{1}{L} \cos^{-1}\left[\frac{1}{2}(A + D)\right]$$

⇨

$$\left|\left[\frac{1}{2}(A + D)\right]\right| \leq 1, \quad K \text{ is real}$$

$$\left|\left[\frac{1}{2}(A + D)\right]\right| > 1, K \text{ is imaginary}$$

Identified frequency bands where K is Real and where K becomes complex known as "Forbidden " "Dispersion" or "Brillouin" Zones

Wave Propagation is "evanescent" in Brillouin Zones

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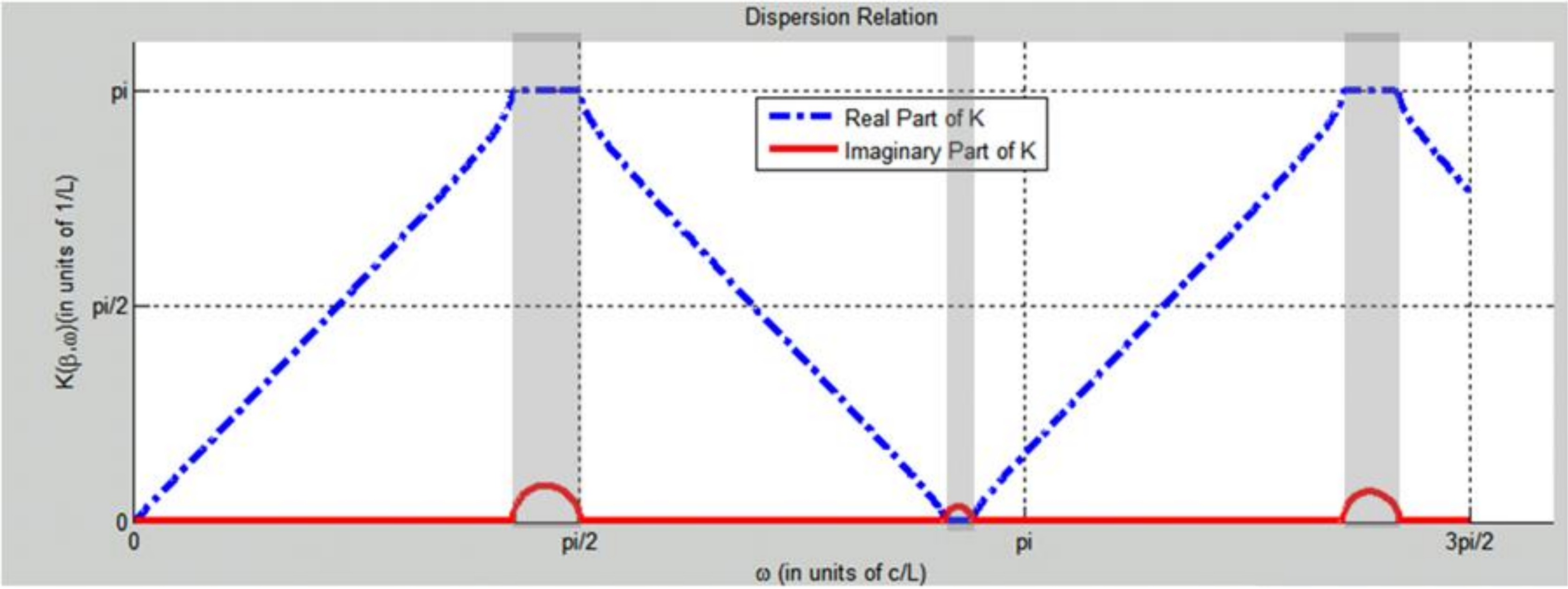
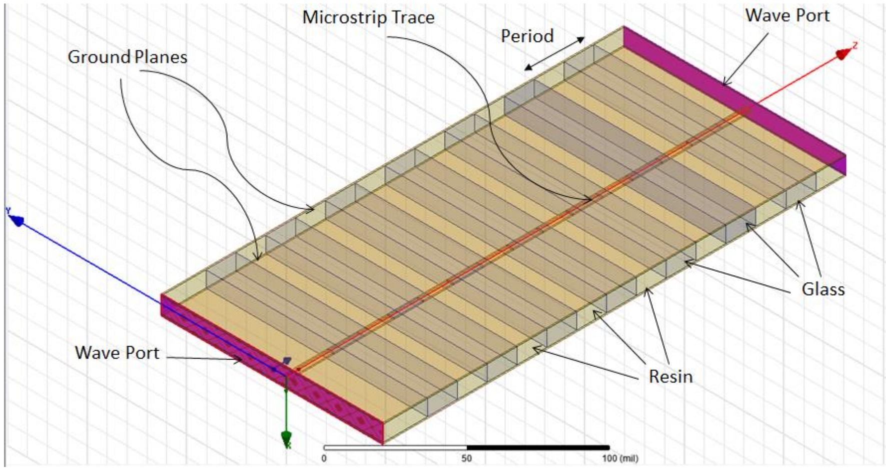
Periodic Wave Propagation: "Brillouin" Zones

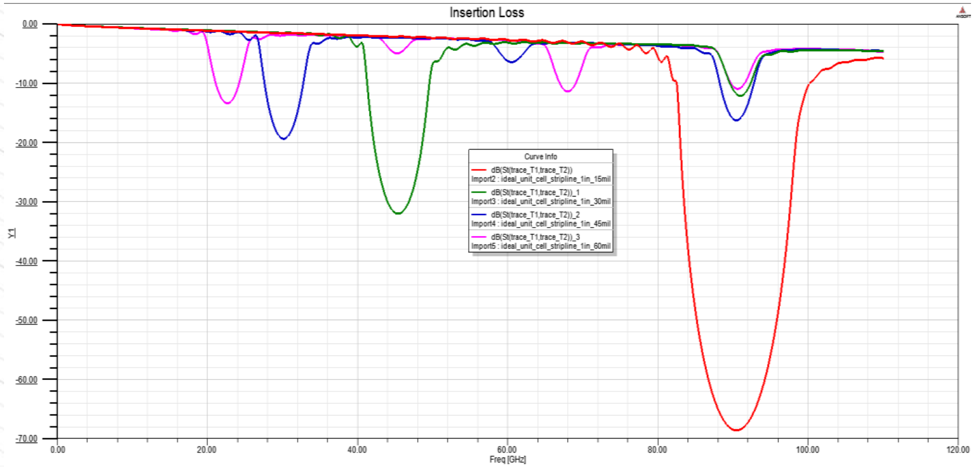


$$K(\beta,\omega) = \frac{1}{L}\cos^{-1}\left[\frac{1}{2}(A+D)\right]$$

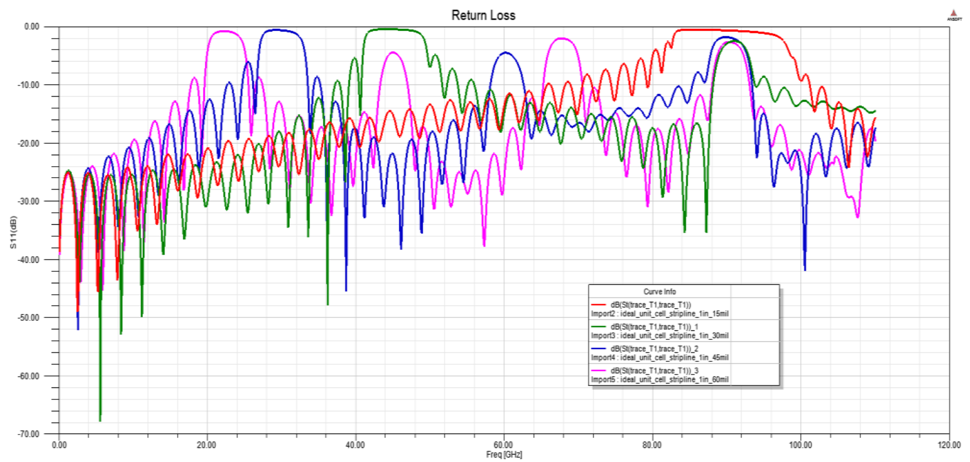
- Normalized Plot of dispersion relation between K and ω for direct incident (β=0)
- Er1=3.5 ; L1 = L/2
- Er2=6.0 ; L2 = L/2
- Shaded regions are the Brillouin Zones where Imaginary part of K is not zero.

Simulations Ideal Periodic Slabs





- Er1=3.5 ; Er2=6.0 ; L2 = L1

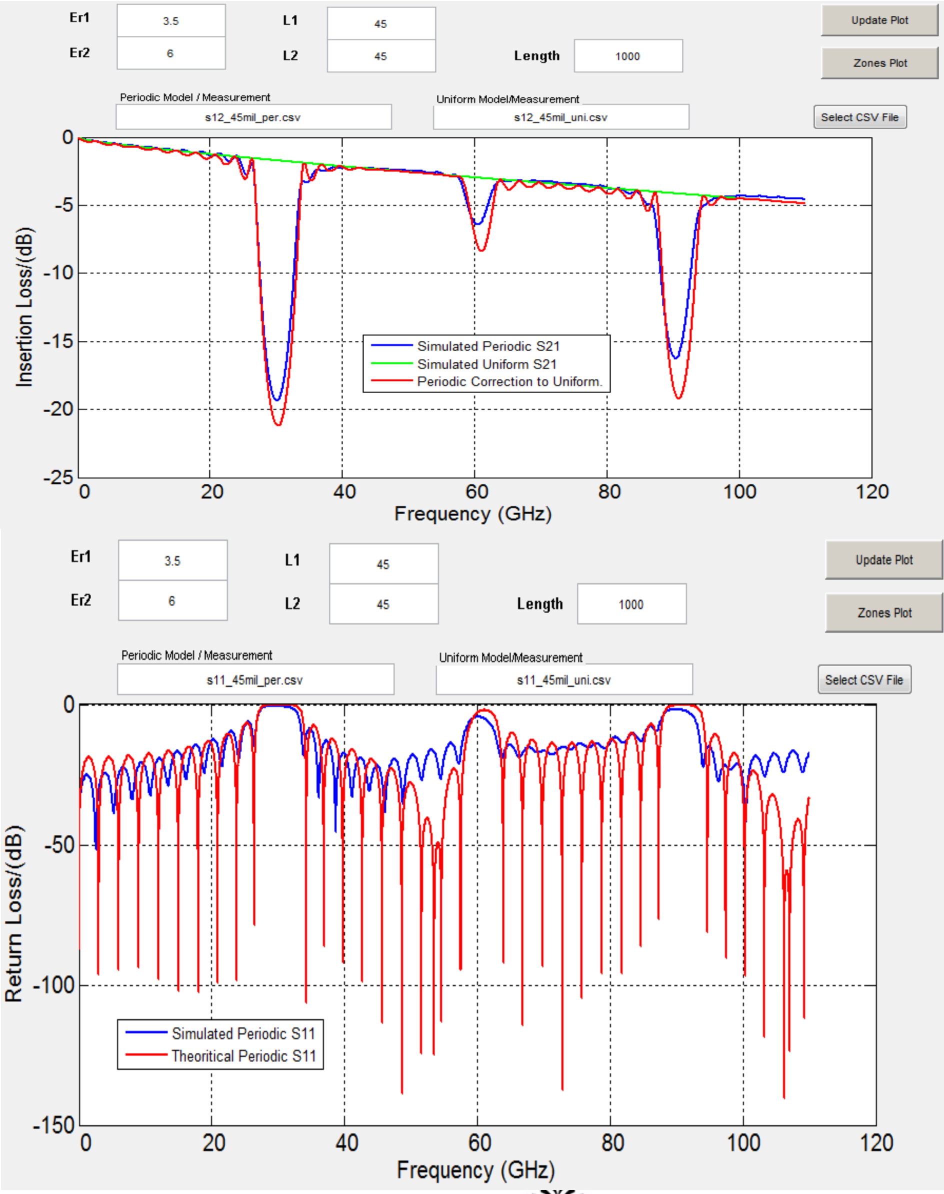


- 3D-EM Simulations of Stripline
- Ansys HFSS Simulations
- Modal S Parameters
- Resonances in Insertion and return loss S-parameters.
- Dependency on Unit Cell size

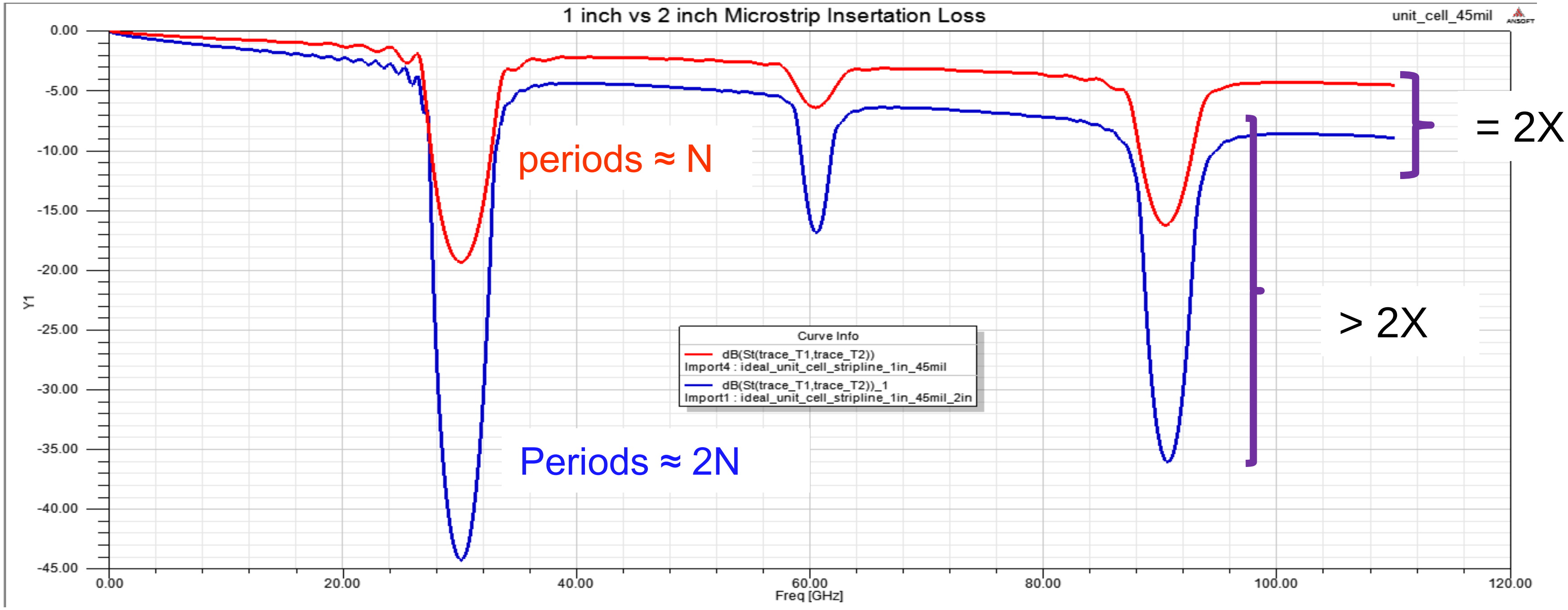
L= 15,30,45,60mil

Ideal Periodic Slabs- Simulations and Theory (Wave Solution)

- 45mill Unit cell Shown
- Modal S Parameters
- Insertion loss matching
- Return Loss Matching



Ideal Periodic Slabs- Dependency on total length



- Comparing HFSS results of 1" line and 2" lines
- S12 in pass bands scales linearly(2x)
- S12 in stop bands scales according to:

Linear scaling of simulation models is not correct!

$$SP_{12} = \frac{2\ \sin KL}{\left\{A + D + \frac{B}{Z_0} + CZ_0\right\}\sin(N)KL + \{A + D\}\sin(N-1)KL}$$

(for Reciprocal line)

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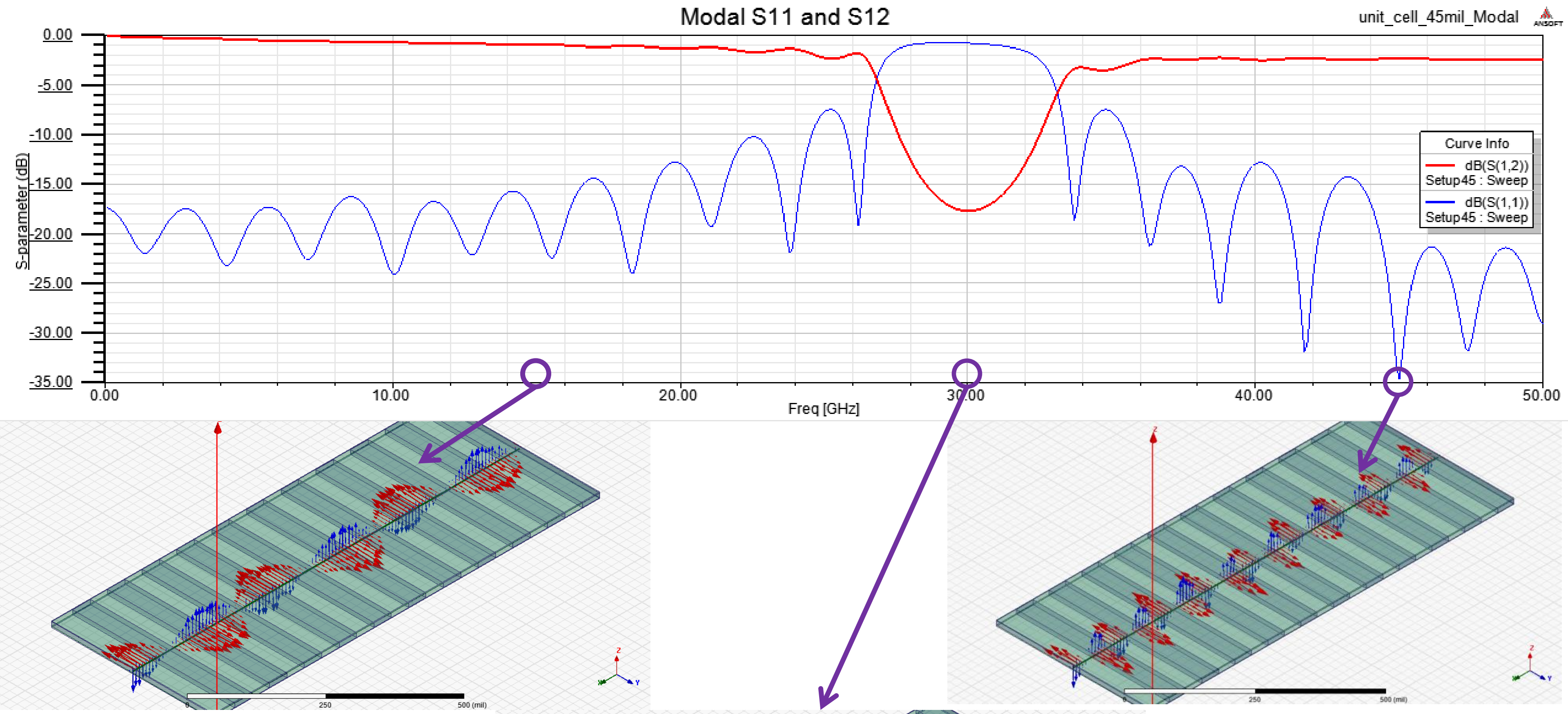
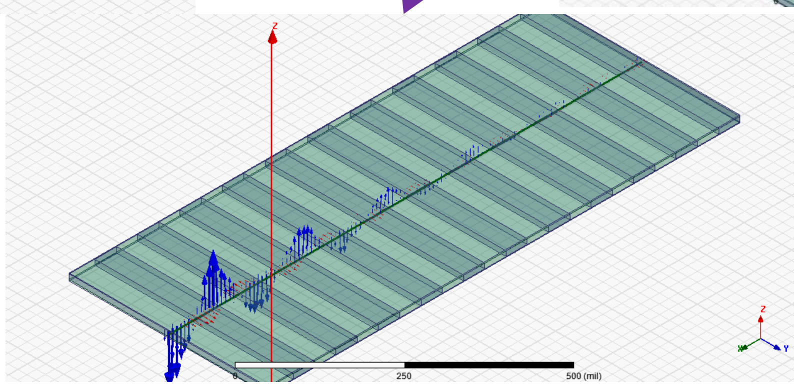
Propagating and Evanescent Bands of a Periodic Medium





Ideal rectangular Slabs
45mil pitch period

E and H fields
on ground plane

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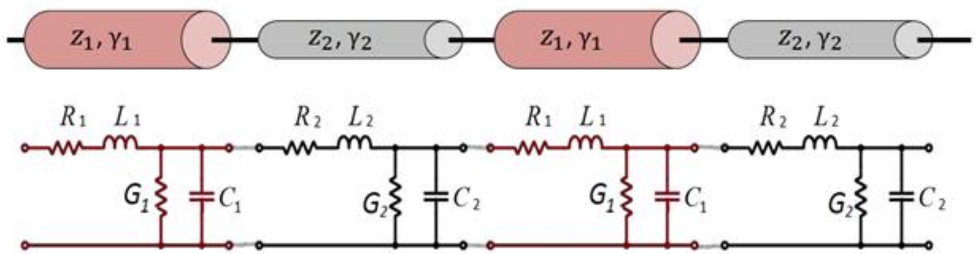
Periodic Transmission Line Analogy (method #2)

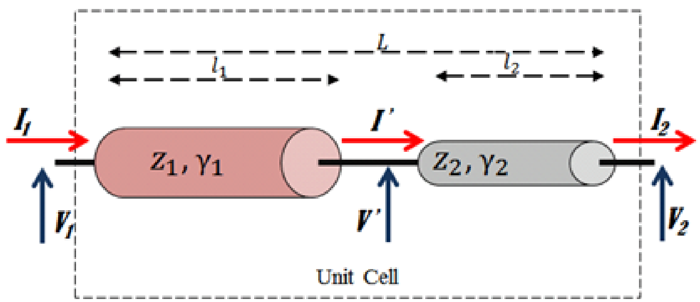


$$Z_1 = \sqrt{\frac{R_1 + j\omega L_1}{G_1 + j\omega C_1}}$$

$$\gamma_1 = \sqrt{(R_1 + j\omega L_1)(G_1 + j\omega C_1)}$$

$$Z_2 = \sqrt{\frac{R_2 + j\omega L_2}{G_2 + j\omega C_2}}$$

$$\gamma_2 = \sqrt{(R_2 + j\omega L_2)(G_2 + j\omega C_2)}$$



$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} \cosh\gamma_1 l_1 & Z_1 \sinh\gamma_1 l_1 \\ \frac{\sinh\gamma_1 l_1}{Z_1} & \cosh\gamma_1 l_1 \end{bmatrix}$$

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} \cosh\gamma_2 l_2 & Z_2 \sinh\gamma_2 l_2 \\ \frac{\sinh\gamma_2 l_2}{Z_2} & \cosh\gamma_2 l_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Define: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$

ABCD matrix of Unit Cell

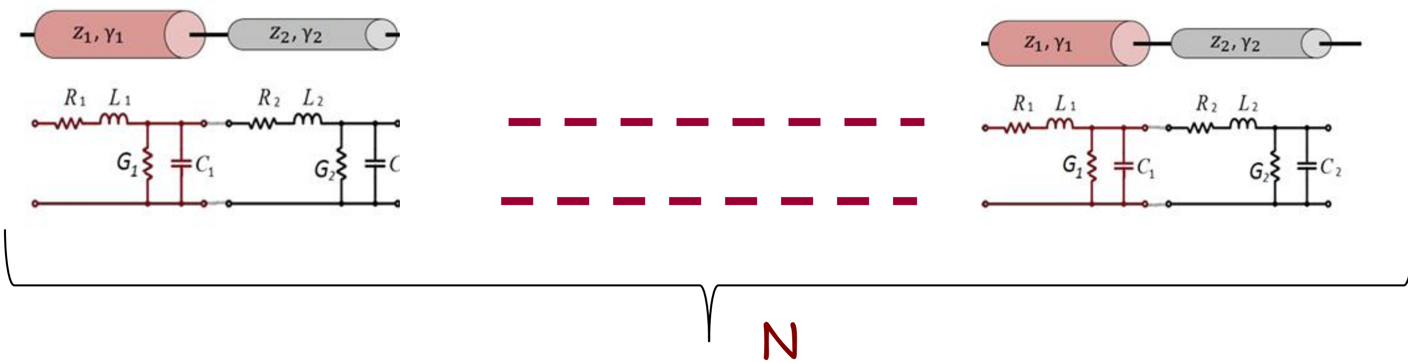
Floquet's result for periodic propagation ⇨

$$e^{\pm 2jkL} - (A + D)e^{\pm jkL} + 1 = 0$$

Periodic Transmission Line Analogy…

$$\frac{A+D}{2} = \cos kL = \cosh\gamma_1 l_1 \cosh\gamma_2 l_2 - \frac{1}{2}\left(\frac{Z_2}{Z_1} + \frac{Z_1}{Z_2}\right)\sinh\gamma_1 l_1 \sinh\gamma_2 l_2$$

→ Brillouin zones



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^N$$

- Dispersion Relation is a function of Characteristic Impedances , Propagation Constant and dimensions of unit cell.
- Unit cell can be characterized using 3D/2D EM simulations or measurements.
- $[ABCD]^N$ matrix can be converted to Terminal S-parameters.
- **Applicable for most practical problems (TEM modes).**

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Cascading Periodic Structures- Chebyshev Identity

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^N = \begin{pmatrix} AU_{N-1} - U_{N-2} & BU_{N-1} \\ CU_{N-1} & DU_{N-1} - U_{N-2} \end{pmatrix}$$

$$U_N = \frac{\sin(N+1)KL}{\sin KL} \qquad \cos(\mathrm{KL}) = \frac{\mathrm{A+D}}{2}$$

$$SP_{11} = \frac{(AU_{N-1} - U_{N-2}) + \frac{BU_{N-1}}{Z_0} - CU_{N-1}Z_0 - (DU_{N-1} - U_{N-2})}{(AU_{N-1} - U_{N-2}) + \frac{BU_{N-1}}{Z_0} + CU_{N-1}Z_0 + (DU_{N-1} - U_{N-2})}$$

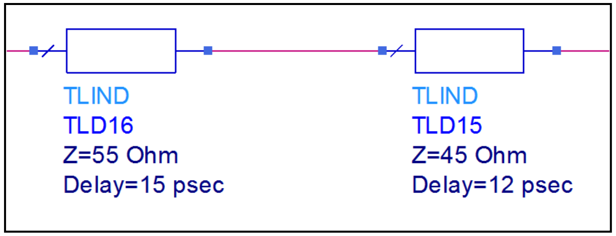
$$SP_{21} = \frac{2}{(AU_{N-1} - U_{N-2}) + \frac{BU_{N-1}}{Z_0} + CU_{N-1}Z_0 + (DU_{N-1} - U_{N-2})}$$

- Compute ABCD matrix of cascaded unit cells.
- Valid For reciprocal networks only. i.e **AD-BC=1**.
- ABCD Matrix to S-Parameter conversion is straight forward. (Pozar)
- Additional Periodic Losses will be an addition in log scale

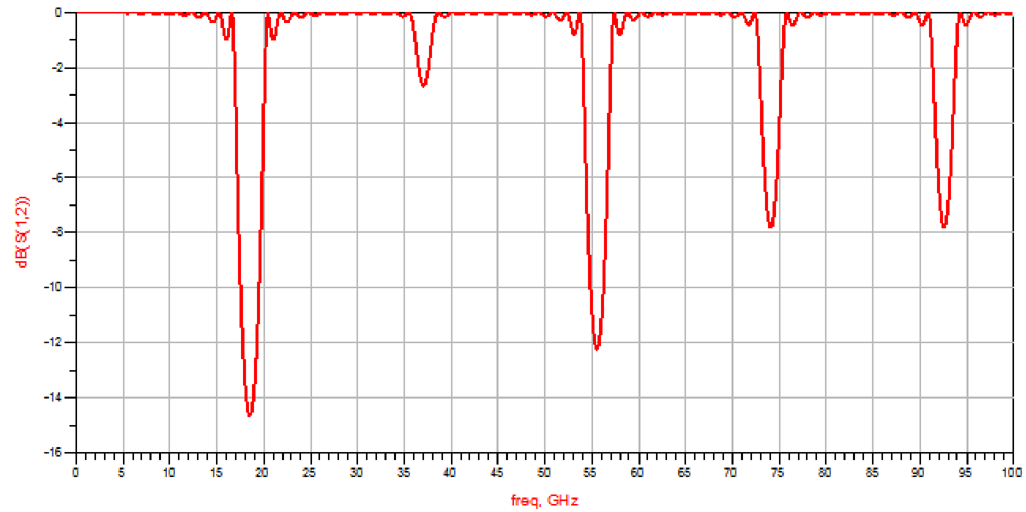
$$S_{21}(\omega) = \left(10^{-\alpha_{dielec.}\frac{l}{20}}\right)\left(10^{-\alpha_{rough}\frac{l}{20}}\right)\left(10^{-\alpha_{impuriy}\frac{l}{20}}\right)\left(10^{-\alpha_{periodic}\frac{l}{20}}\right)$$

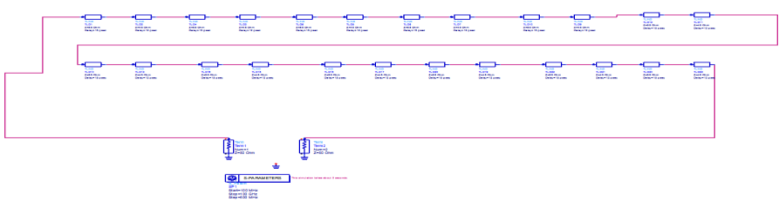
Simulation Verification of periodic T-line theory

Agilent ADS S - Parameter Simulation

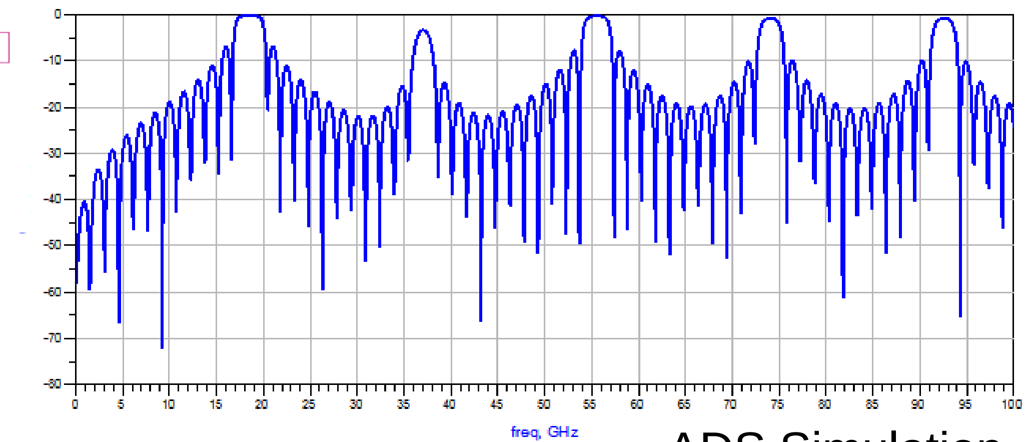


Unit Cell





Simulation setup with 12 unit cells



ADS Simulation

Assumed Loos-less Tlines ➔ Neglect R and G

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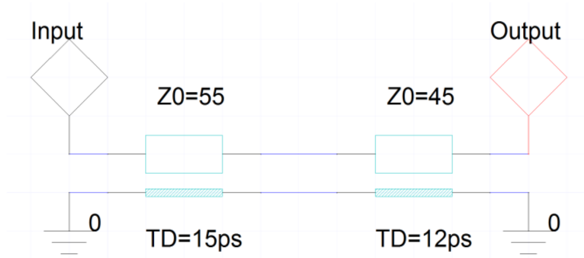
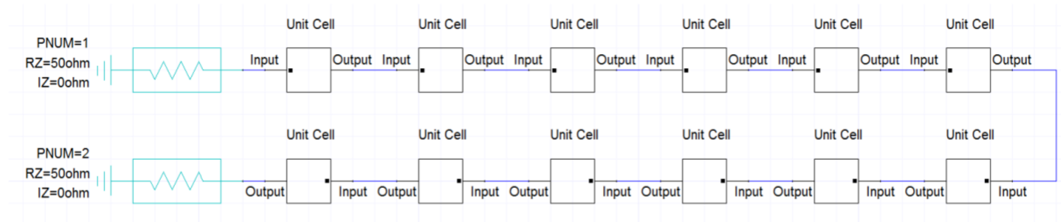
Simulation Verification of periodic Theory

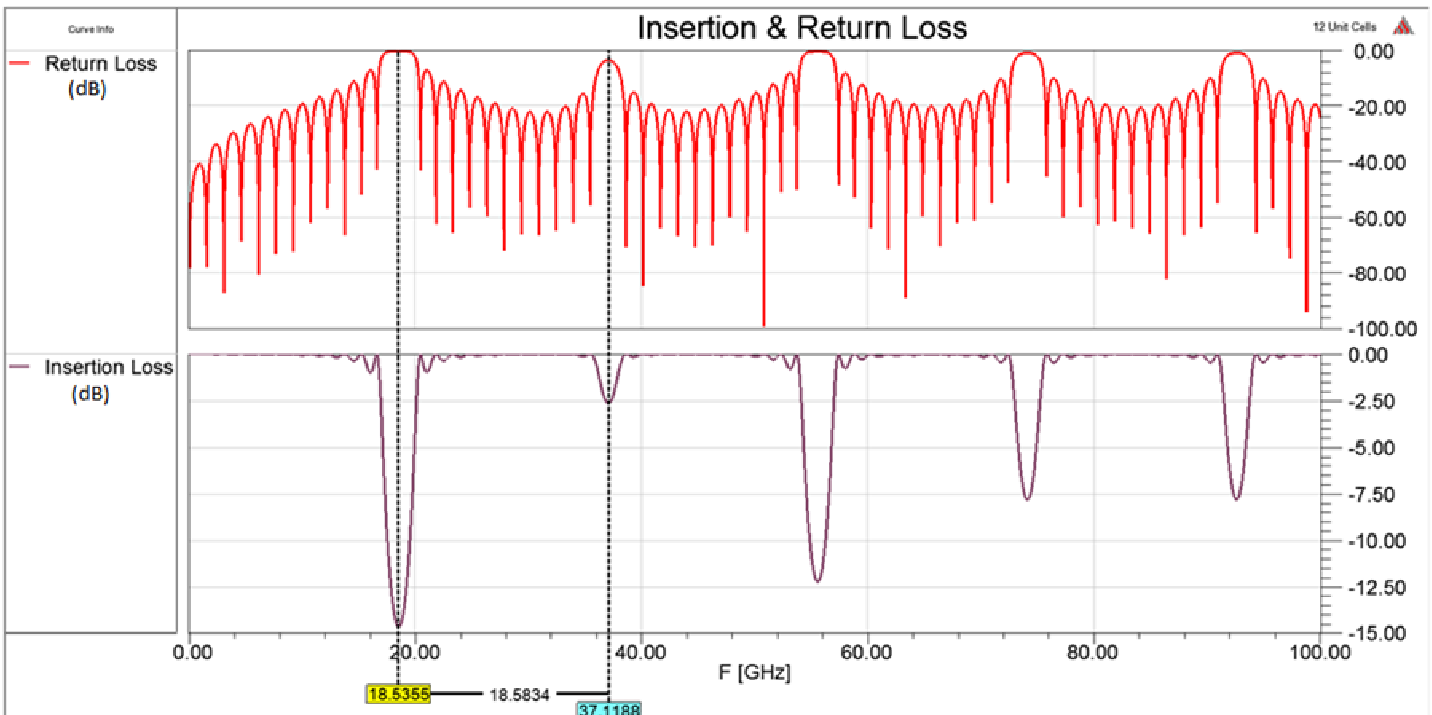
Spice S - Parameter Simulation



Unit Cell



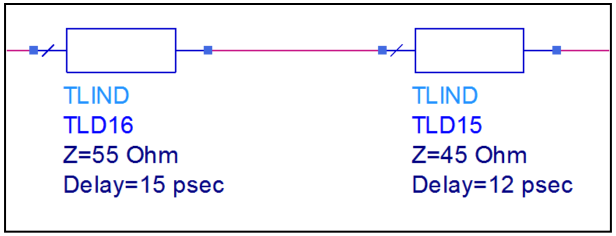
Simulation setup with 12 unit cells



Simulated S11 and S12

Analytical Verification of periodic T-line theory

Analytical Computation of S12



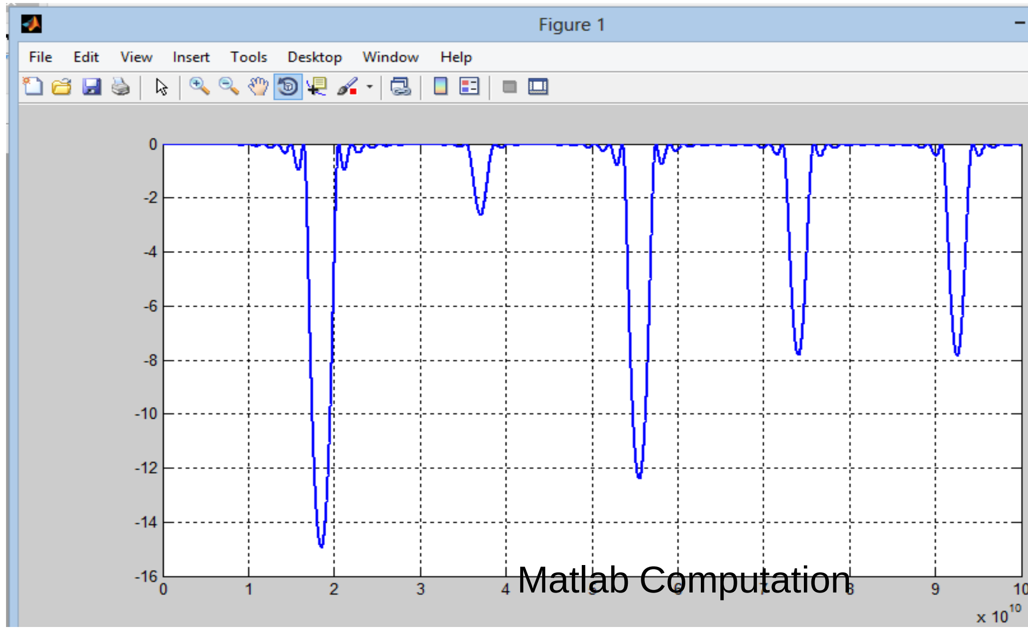
Unit Cell

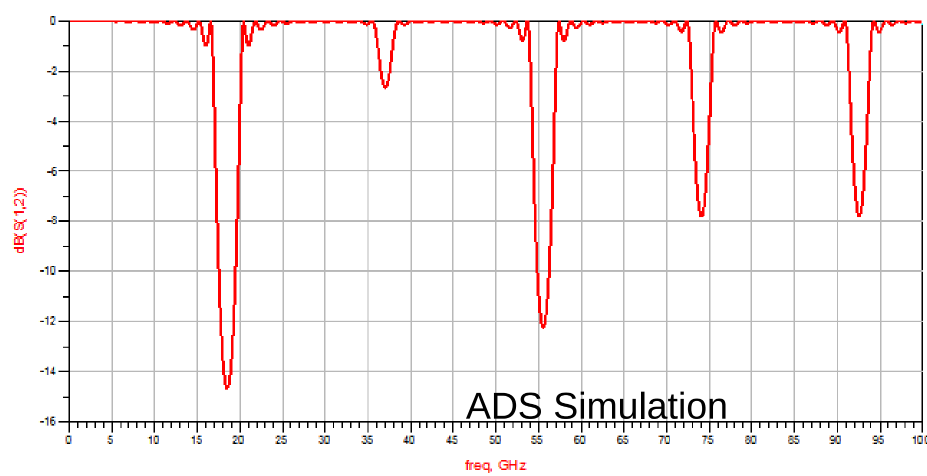
$$Z_1 = \sqrt{\frac{R_1 + j\omega L_1}{G_1 + j\omega C_1}}$$

Assumed Loos-less Tlines ➔ Neglect R and G

L1 = T1*Z1 = 825pH/m
C1 = T1/Z1 = 2.72E-13 F/m

L2 = T2*Z2 = 540pH/m
C2 = T2/Z2 = 2.67E-13 F/m





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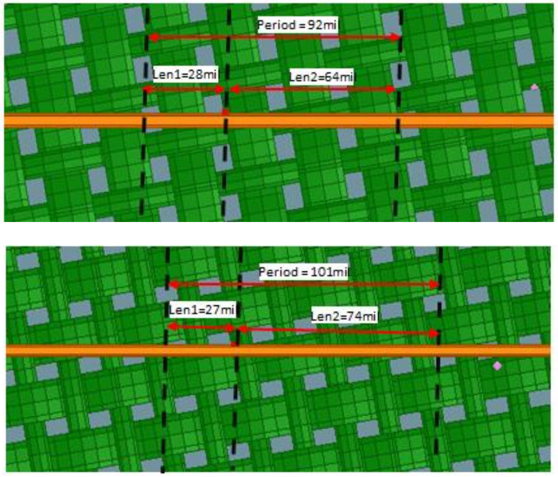
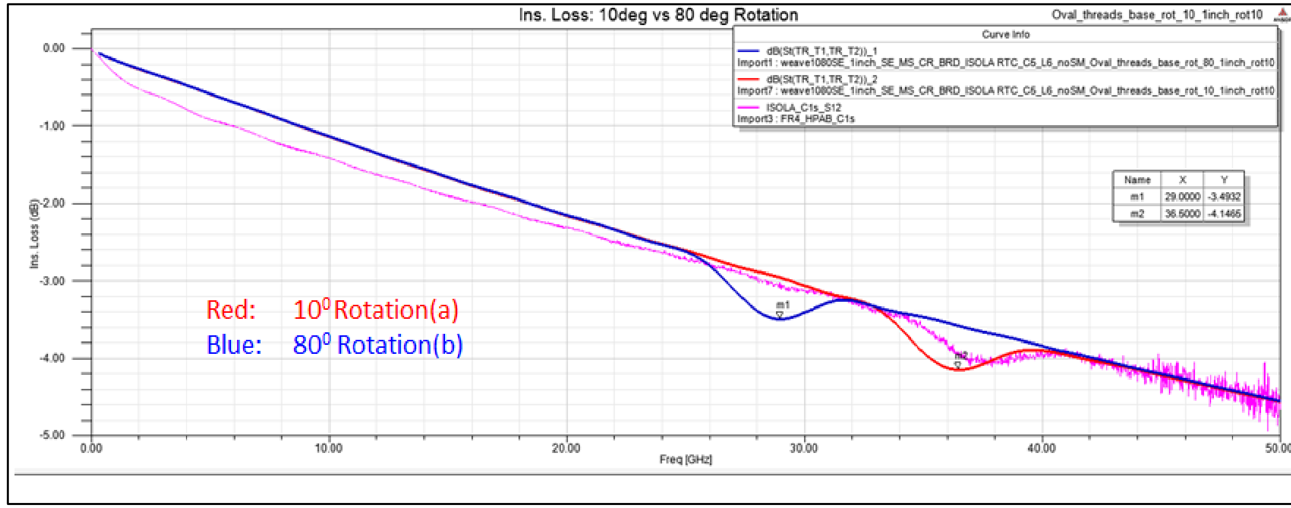
Applications to SI problems: Fiber Weave





- Rotating traces w.r.t fiber orientation is a common practice
  - ✓ Could help skew problem
  - ✓ Loss problem still present
- 1" Microstrip Trace on Type 1080 Weave, 10^0 & 80^0 Rotation simulated
- Simulation model Stackup Parameters matched to "ISOLA RTC1" test Board
- 10^0 rotated simulations matched to measurement

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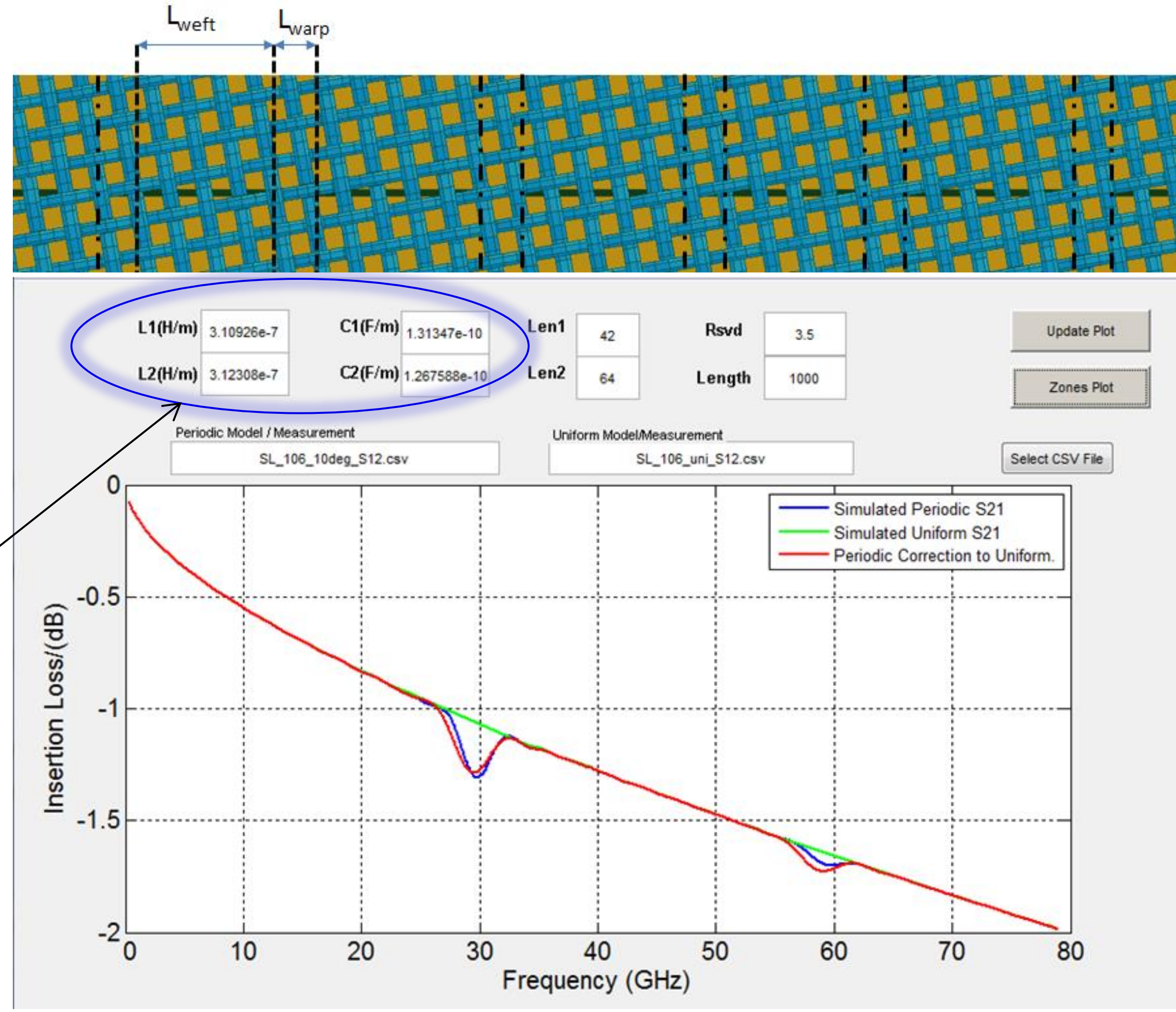
E.g. Simulation and Theory

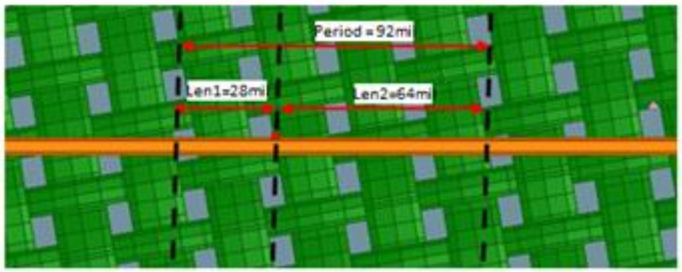
- Type 106 Weave
- 1" Stripline Trace
- 10^0 Rotated
- MCP Graphically Measured
- Periodic Transmission Line Based Approach
- Unit-Cell L & C from 2D Field Solver

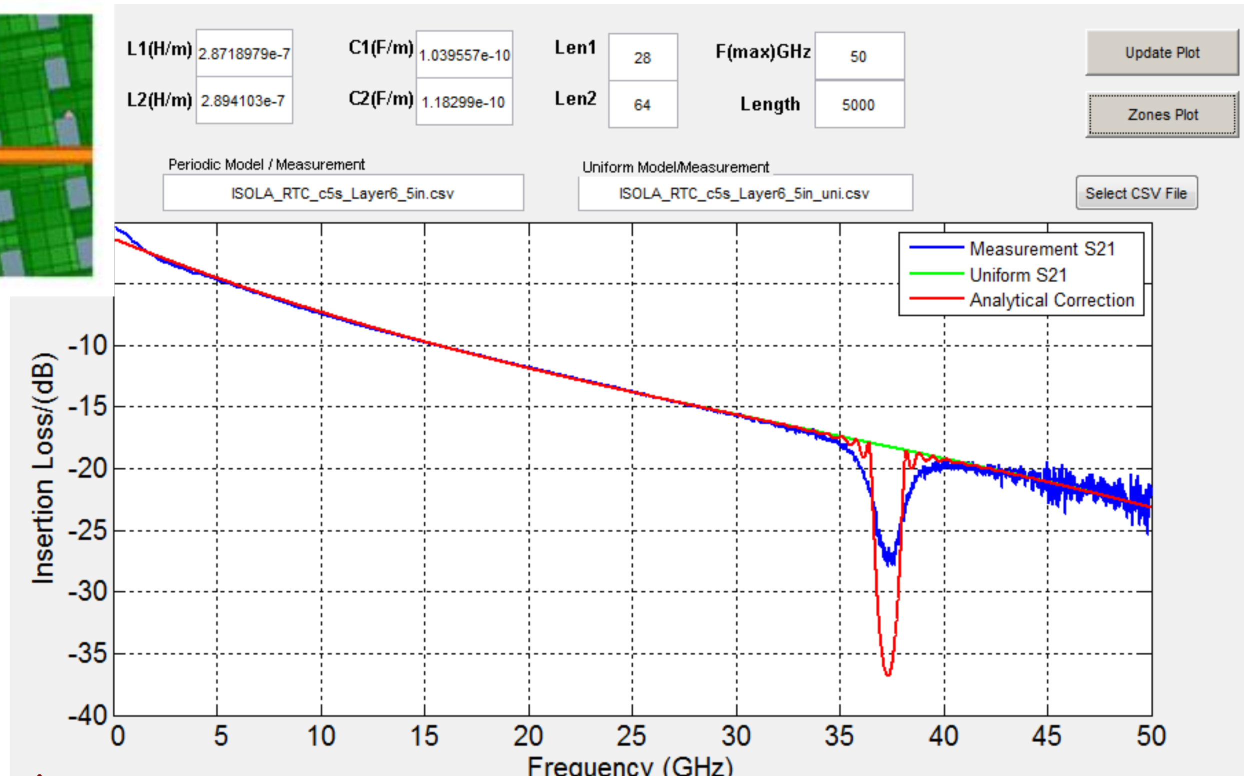


T-Line Unit-Cell Periodic Theory matched to Simulations

T-Line Analogy recommended for complex, real-world applications

E.g.: Measurement and Theory





- "ISOLA RTC1"
- 5" Microstrip Trace
- Type 1080 Weave
- 10^0 Rotation

T-Line Unit-Cell Periodic Theory

L,C Parameters of Unit cell computed using 2D-Field Solver

Acceptable level of correlation

Mismatches are expected due to: - Measurement accuracy

- Imperfections in real fiber weave

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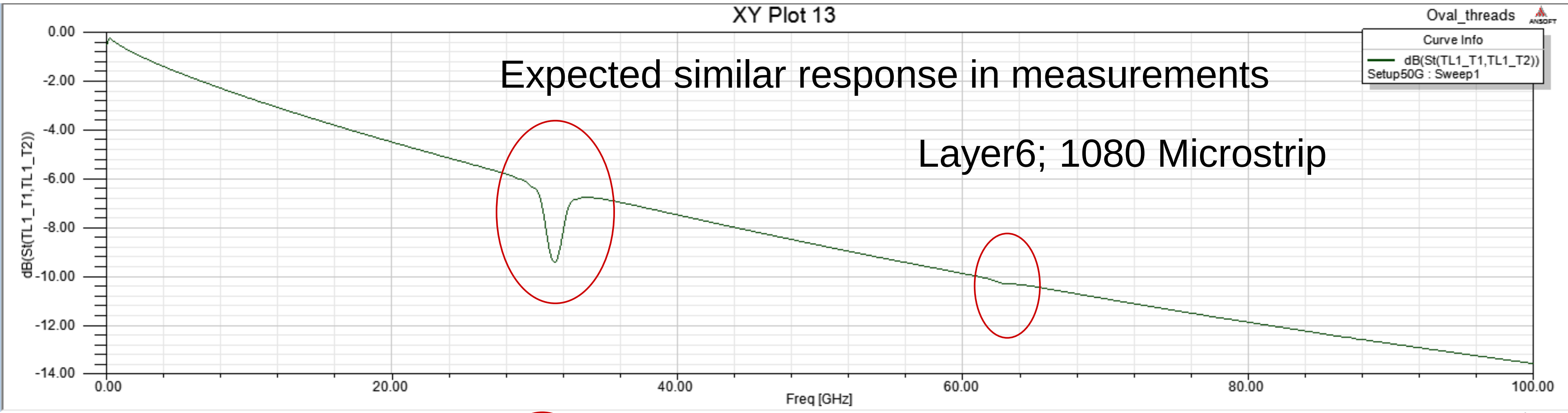
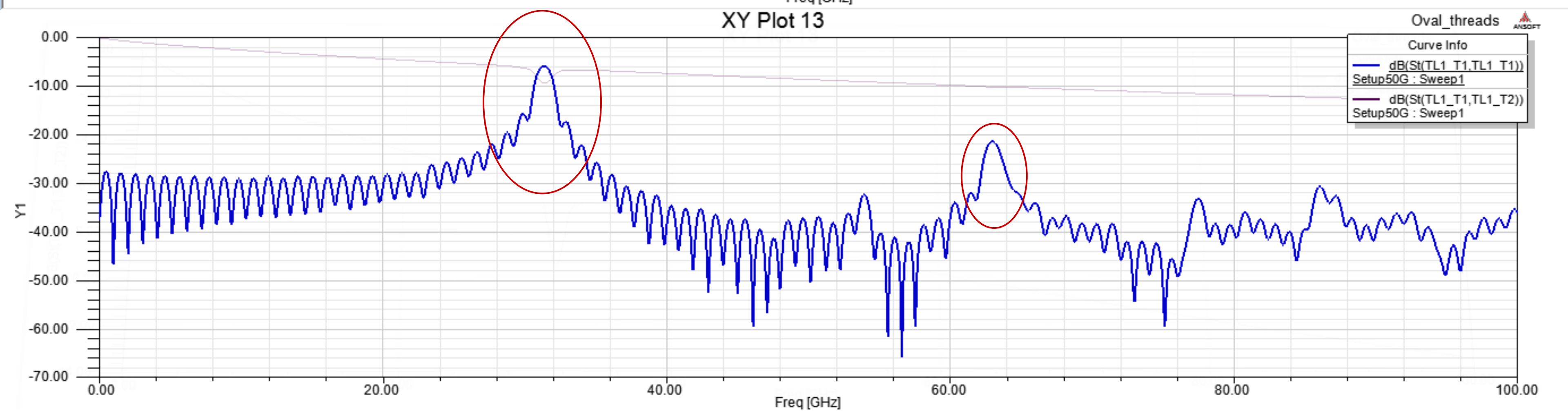
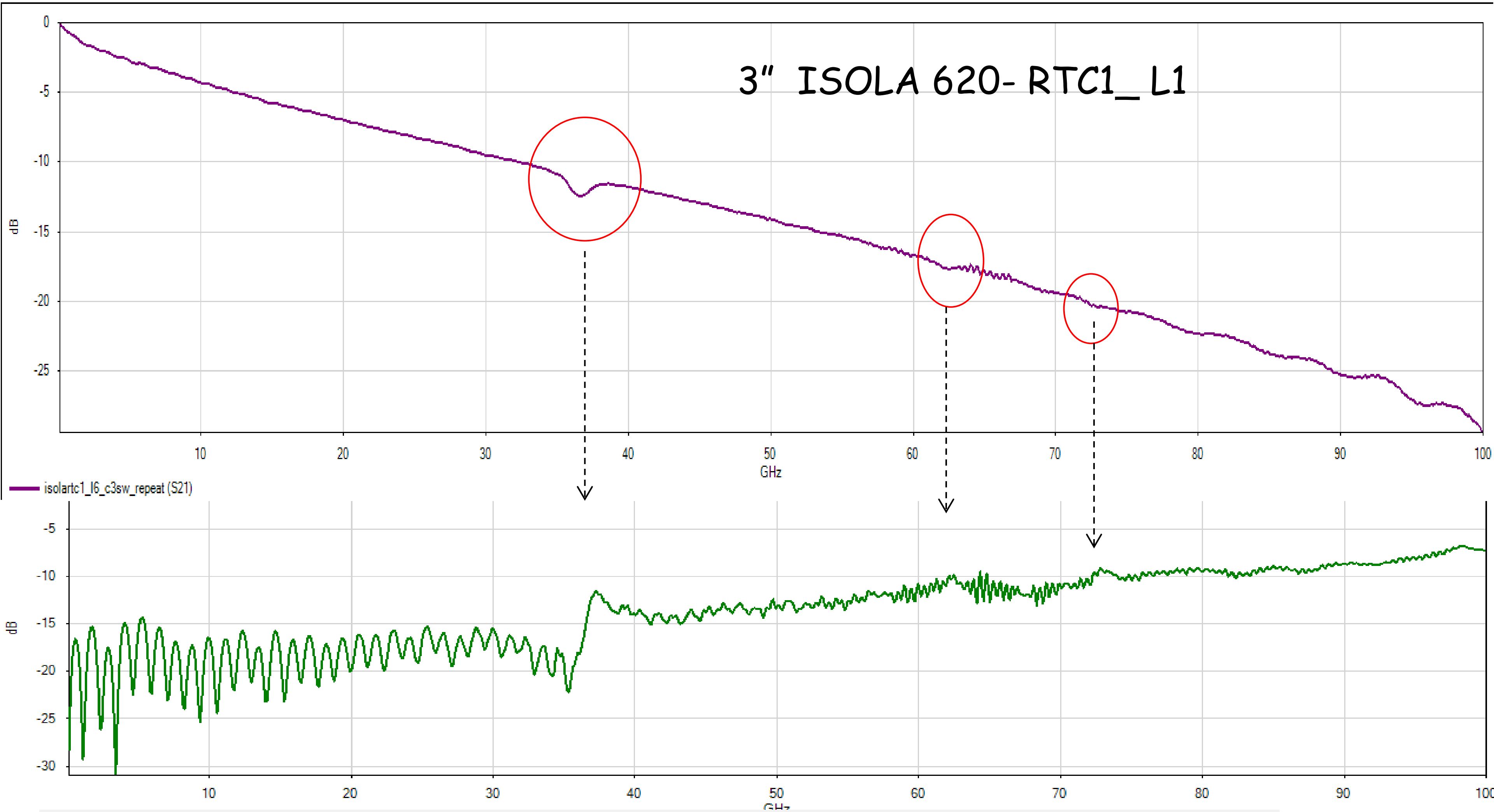
High frequency loss: Simulations





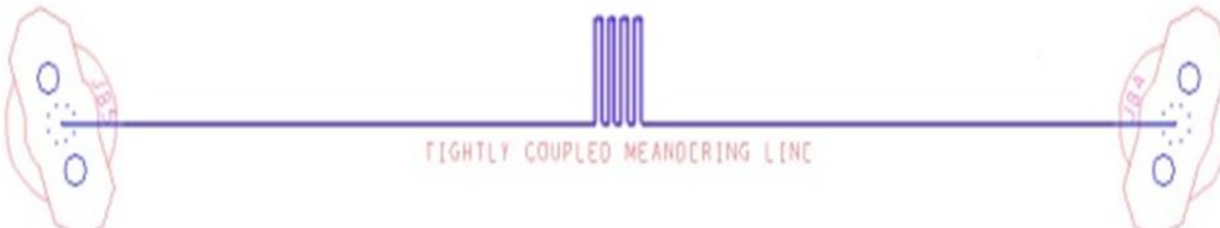
- 3” ISOLA RTC 1 board modeled in HFSS and simulated
- Expected measurements to be similar to simulations
- 2nd and 3rd resonances are expected to be small.

Measurements- ISOLA RTC 3" MS Line

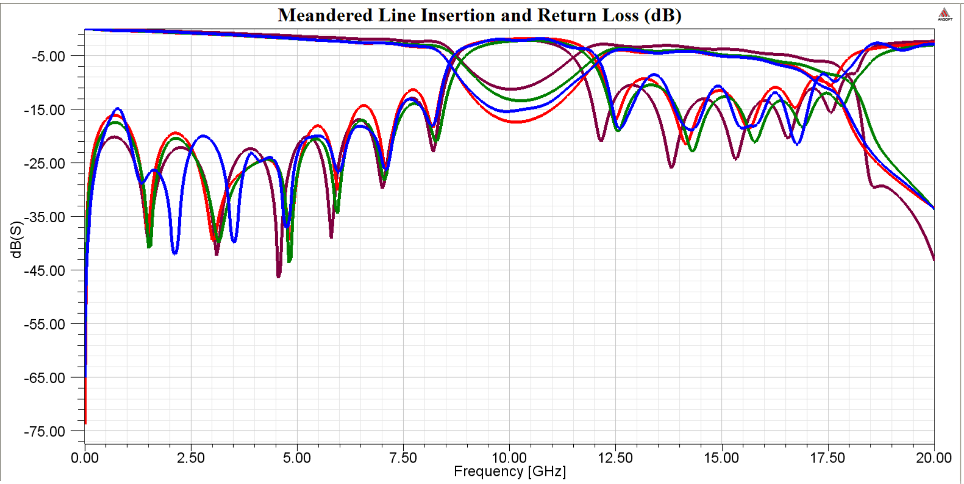


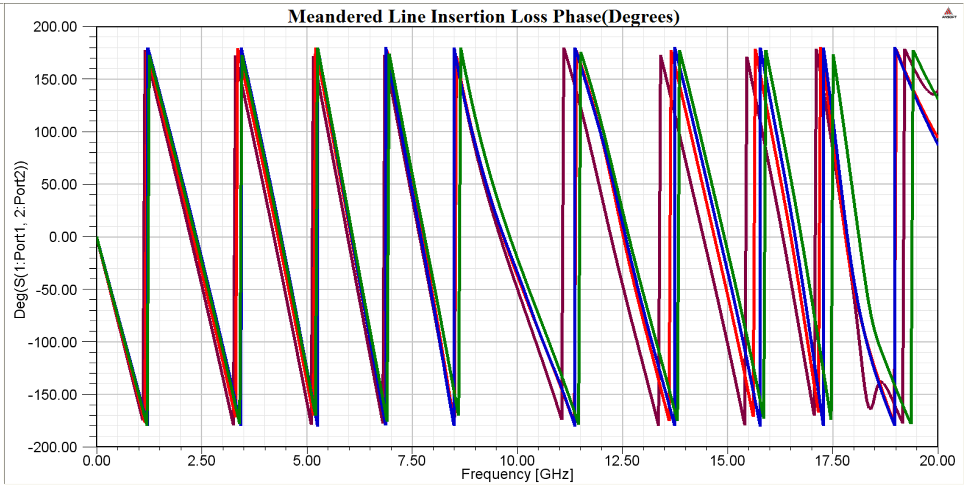
- Limited highest frequency sweep to 100GHz
- Limited Measured lengths to 3" ➔ improved dynamic range

Meandered Microstrip



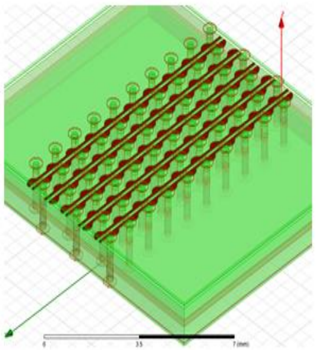
3D Full-Wave FEM **Hybrid Full-Wave** **Measured** **MoM**

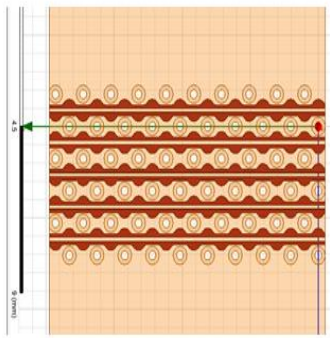


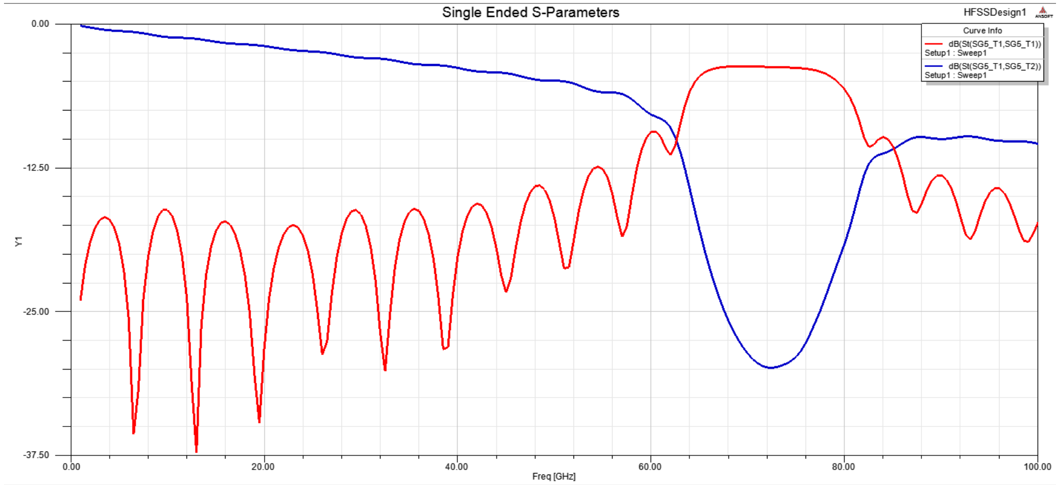


Simulation Method	Elapsed Time	Peak Memory	Total Cores
Hybrid	11 s	54 MB	4
3D FEM	1hr 30m	1.79 GB	4
3D MoM	17m 53s	213 MB	4

Applications to SI problems: Pin Field Routing







- Rotating through a pin-field can cause periodic discontinuity.
  - Overall impedance profile can be improved
  - May cause deep resonances at higher speeds.
- Approximate attenuation regions can be analytically predicted by analyzing a unit cell only

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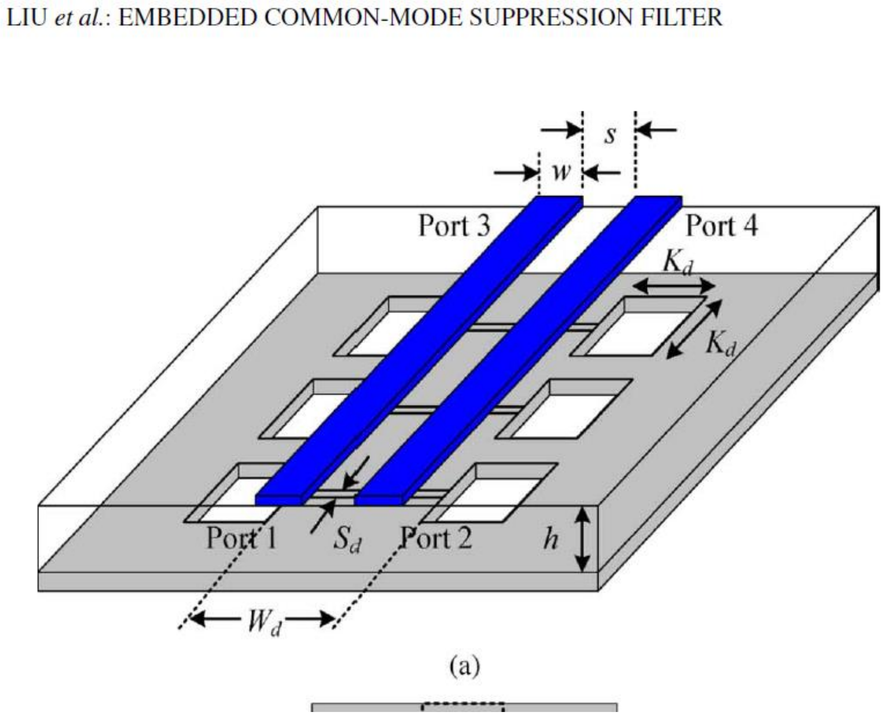
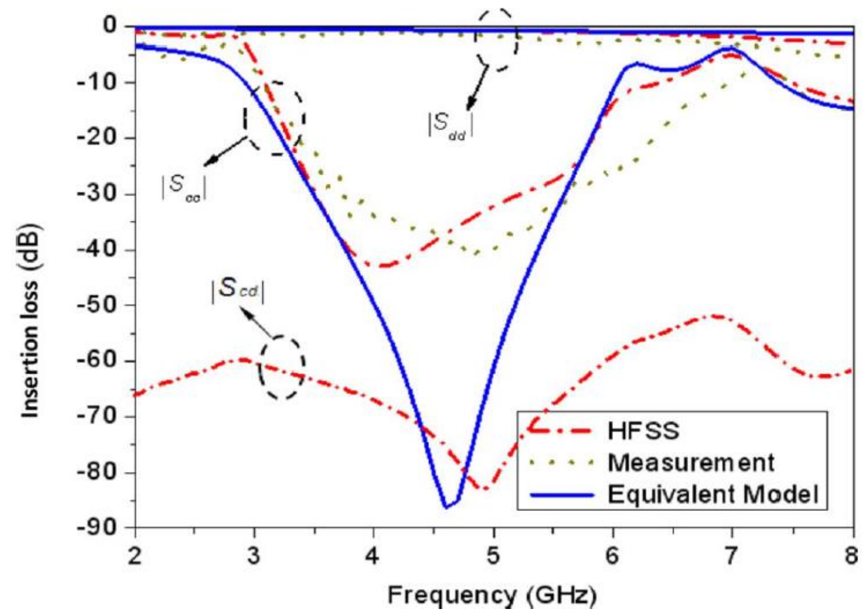
Applications to SI problems: EBG Filters





- Precisely designed band gaps can be used to suppress certain noise frequencies
- E.g.: An embedded common mode suppression filter using periodic ground voids

- Source: Liu, Wei-Tzong, Chung-Hao Tsai, Tzu-Wei Han, and Tzong-Lin Wu. "An Embedded Common-Mode Suppression Filter for GHz Differential Signals Using Periodic Defected Ground Plane." IEEE Microwave and Wireless Components Letters 18, no. 4 (2008): 248-50.

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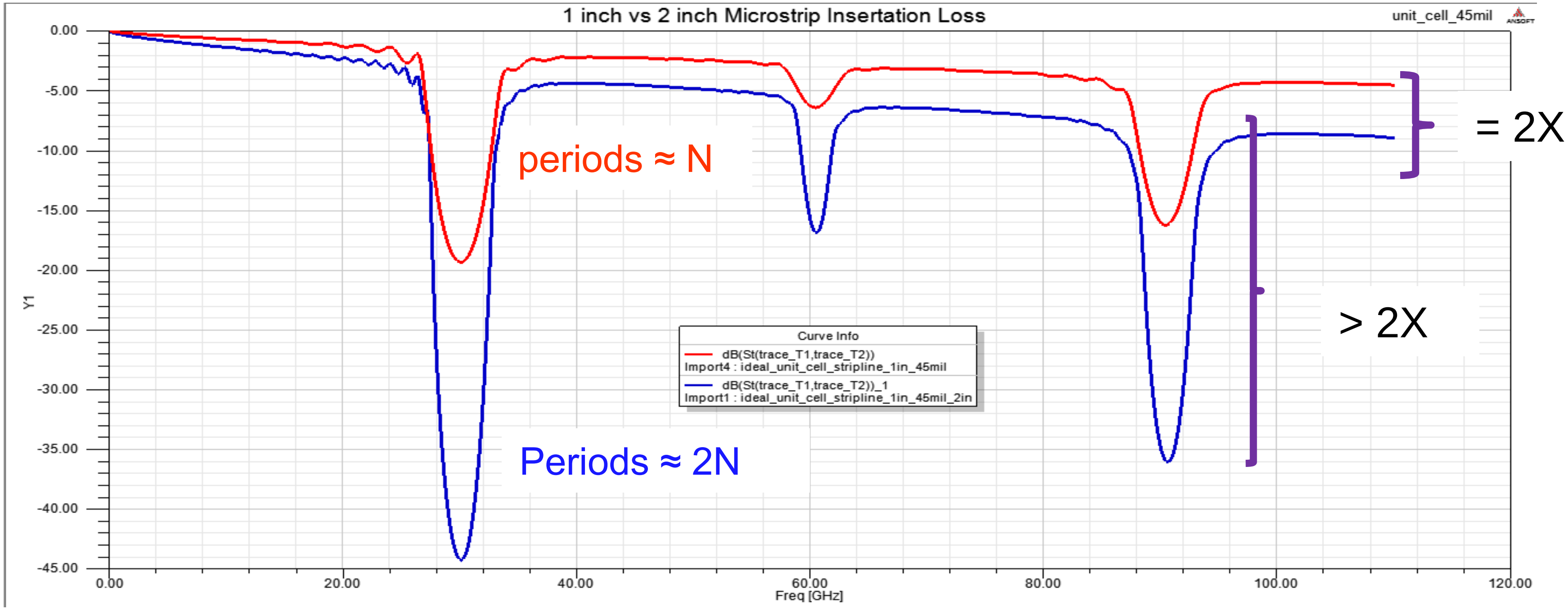
Ideal Periodic Slabs- Dependency on total length



- Comparing HFSS results of 1" line and 2" lines
- S12 in pass bands scales linearly(2x)
- S12 in stop bands scales according to:

Linear scaling of simulation models is misleading!

$$SP_{12} = \frac{2\ \sin KL}{\left\{A + D + \frac{B}{Z_0} + CZ_0\right\}\sin(N)KL + \ \{A + D\}\sin(N-1)KL}$$

(for Reciprocal line)

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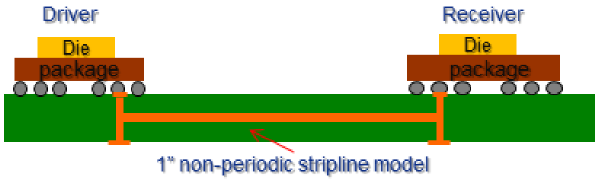
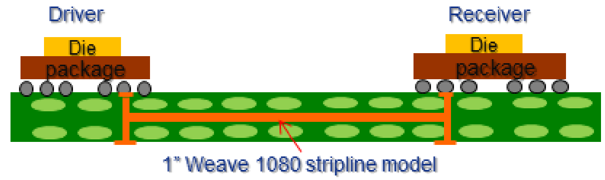
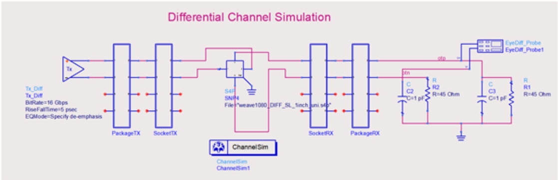
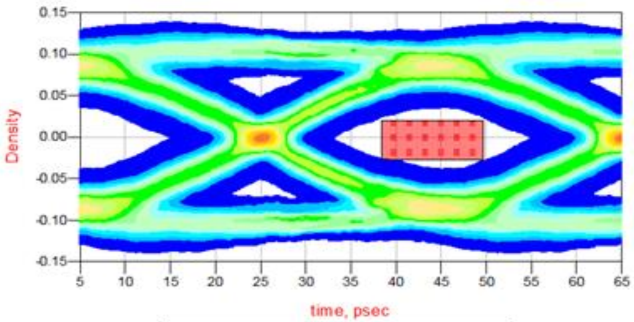
Periodic Loading impact on system eye margins

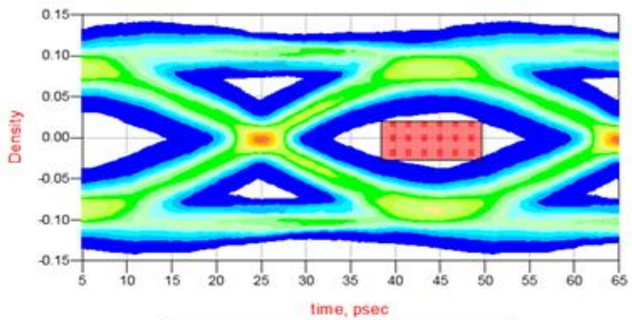




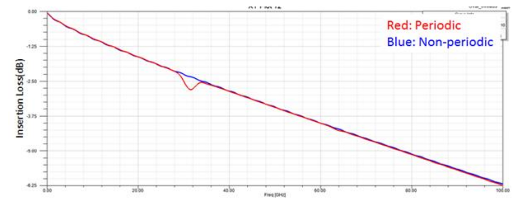




measurement	Summary
Level1	0.085
Level0	-0.085
Amplitude	0.170
Height	0.079
Width	2.720E-11



measurement	Summary
Level1	0.084
Level0	-0.087
Amplitude	0.170
Height	0.074
Width	2.736E-11



- Simulated a 1 inch Stripline Differential channel to emulate a 16GT/s system.
- Removing Periodic loading could improve the eye by over 5mV per inch.
- Expect greater improvements on longer channels.
- Accounting for Periodic Loading is important for a good system design

Summary

- Periodic discontinuities are common in interconnects.
- Periodic resonance impacts loss performance at higher frequencies.
- Solutions using 2 methods
  - Solutions to Wave equations
  - Transmission Line analogy
- T-line method is preferred for practical SI problems
- High frequency channel performance will be affected by periodic effects
- Proposed analytical technique can be deployed in tools for modeling PCB traces with fiber weave effects